MATHEMATICS TEACHING

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MATHEMATICS TEACHING

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NUMERACY—

An Important Contribution

THE LANGUAGE OF MATHEMATICS is an attempt to provide in a compelling yet substantial form an insight into the nature of mathematics. It is designed to give to Sixth Form pupils and Training College students a taste of — and for — the intellectual pleasures which mathematics affords.

In both content and appearance it is entirely different from the conventional text-book. It has been designed by the distinguished typographer Will Carter and the two-colour illustrations by Margaret Clark are of outstanding quality.

The author first deals with numbers and how they evolved and with our notation and units in their historical setting. He then goes on to consider compound units, space and elementary geometry, algebra and statistics. The shapes of marine animals, the number patterns to be discerned in flowers, the shape of the Jodrell Bank reflector, the distribution of intelligence, the orbits of artificial satellites — such are the widely diverse materials which he uses in developing his argument.

A booklet of questions to accompany the text volume has been prepared. This provides practical work on the majority of chapters and it will be printed in small editions so that highly "topical" questions on science, economics and other fields of study can be incorporated from time to time.

THE LANGUAGE OF MATHEMATICS

By FRANK LAND

Senior Lecturer in the Department of Education, University of Liverpool

276 pages. Booklet of Exercises 2*s.* 6*d.*
Publication November 1960. Students' Edition 15*s.*

JOHN MURRAY 50 ALBEMARLE STREET LONDON W1

EDITORIAL

THE THREE YEAR COURSE

Before our next issue appears we shall have commenced a fresh academic year. As far as the teaching profession is concerned it will be a notable one, for commencing in September there will be the first three year training college course in this country. To undertake such a step at a time when there is a shortage of teachers is to emphasize the bold and imaginative nature of the venture; it is one which should be approached with responsibility by student and tutor alike, for it depends for its success upon each of them.

In cases such as this, where a course is lengthened, the inevitable question is what shall be done with the extra time, and almost as inevitably there arises a difference of opinion. In this particular instance the choice appears to be fairly well defined: shall the increased time be devoted to an increase in academic studies, or shall it be used to give a fuller training in the art of teaching? From a survey appearing in our current issue, however, it appears that most colleges are choosing the former alternative, and one must question whether this is necessarily a good thing. The study of education as an art or science in itself has always been underrated; it is as though we are afraid to admit to having to learn how to do our job, and as though we should know how to teach without having to learn; in fact, 'anyone can teach' has been the general attitude. In England the possession of a degree in any subject has been a recognized qualification for teaching, and while many good teachers possess no other professional qualifications, one has to ask might they not be more effective if they had undertaken a course of study and training in their craft?

It is right, of course, that the three year training college students should have studied mathematics at a level considerably beyond that to which they will be required to teach, and that they should experience the mental discipline of undertaking advanced study. It is equally essential that they should study deeply the psychological principles involved in imparting knowledge and the effective presentation of the learning situation, and be familiar with the latest ideas on teaching method and material. The courses at many training colleges already fall short in the latter respects, and when the extended course is being planned these matters should not be ignored while pure academic studies are extended.

The matter of school practice, too, calls for urgent consideration. What is needed here is a closer liaison between the colleges and the teachers in the schools. It should be possible to arrange that at least one teacher in a school which receives students can act as tutor to the students while on teaching practice of, perhaps, a term's length. In that time it would be his task to see that the student learnt to put to practical ends the principles taught in the lecture room. It would be necessary, of course, to ensure that this teacher had a thorough knowledge of educational theory and his position would be what one might term an 'extra-mural' training college tutor. Such a person could be extremely helpful, also, in the probationary year.

Much more could be written along these lines, but the essence of the training college course in mathematics should be to develop in the student a true love of learning, an understanding of the objectives and principles of education, an appreciation of the relationship between mathematical thought and psychological perception, a training in observation and experiment in the art of teaching and a desire to teach the subject in a challenging and stimulating manner.

T. J. FLETCHER
PRESIDENT OF THE ASSOCIATION 1960

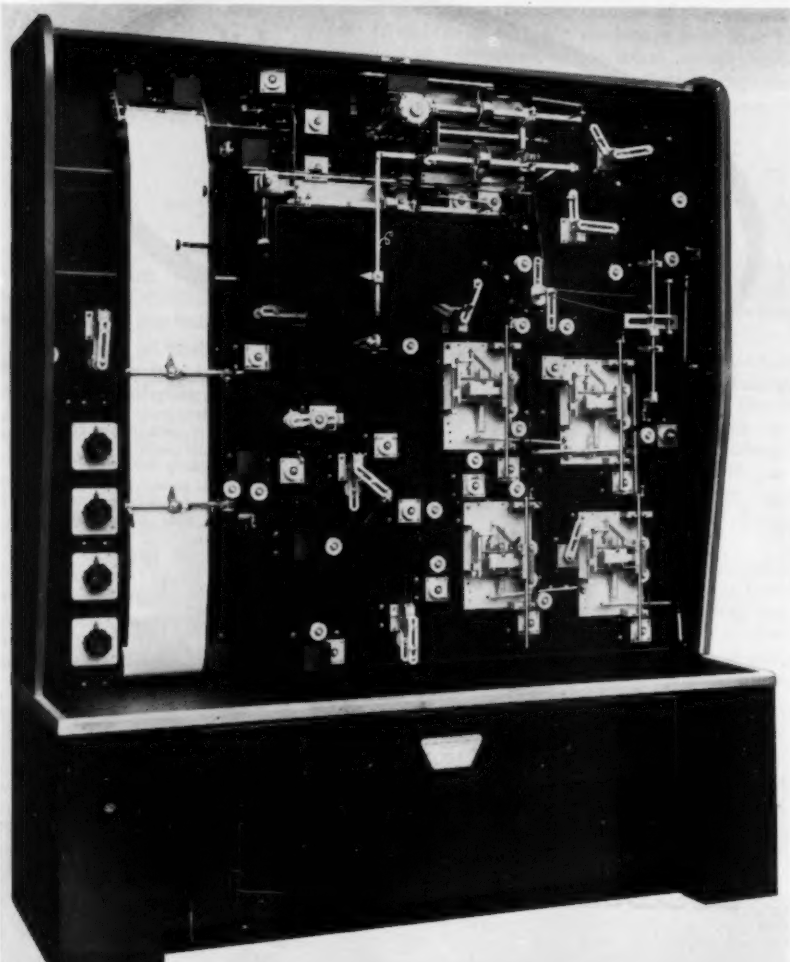
Trevor Fletcher can need little introduction (at least as a name) to readers of this magazine. It was through his enthusiasm that the first issue was produced, and he edited the first nine issues in such a way as to firmly establish it as a leading mathematical journal. He has been a frequent contributor since.

Educated at Beckenham and Penge County Grammar School, he went with a Postmastership to Merton College, Oxford to read mathematics, and graduated after a break during the war when he was working at Malvern on radar. Three years teaching in public schools convinced him of the folly of much current practice and brought the realization that we must find some way of presenting the subject as a main thread in Western culture and not as a bundle of tricks to be used to pass examinations. Since 1950 he has been a lecturer at Sir John Cass College, but although teaching University work nearly all the time he has retained a strong interest in the problems of teaching at all levels.

He attended the second committee meeting of the Association, and has been deeply involved in its affairs ever since. Apart from his work as Editor, he has been an active committee member who has played an invaluable part in guiding the Association through its early years, and also founded the Film Unit; the five mathematical films which he has made have been shown in many parts of the world, and he is now acknowledged as one of the foremost authorities on mathematical films.

He is interested in music and is a keen mountaineer. He is married to another mathematician and very recently they have proved to their satisfaction that one plus one equals three!

Mathematics Teaching is particularly honoured to welcome to the Presidency one who has been so closely connected with the magazine. Our readers will know that in his writings are to be found depth, breadth, thoroughness and inspiration. As President of our Association he cannot fail to inspire us all with his own standards and ideals.



The Air Trainers Link Computer

A MECHANICAL ANALOGUE COMPUTER FOR EDUCATIONAL PURPOSES

A mechanical analogue computer, specially intended for educational uses, has been designed by Air Trainers Link, Ltd., of Aylesbury, Bucks., in collaboration with Professor A. Porter and Dr. J. H. Barker of the Imperial College of Science and Technology, London University. The firm also manufacture a range of components from which it is possible to build a range of smaller computers, or to extend existing machines. Construction kits are much less expensive to buy than a fully assembled computer. The following article is selected, by kind permission of Air Trainers Link, from the technical brochure which they publish. Major sections of this were written by Dr. Barker.—EDITOR.

INTRODUCTION

The device here described is a general purpose mechanical analogue computer ("MAC") in which numbers are represented by linear displacements. It consists of basic units which stimulate the mathematical operations of addition (and subtraction), multiplication (and division), and integration. In the 'multiplier', for example, the imposed straight-line displacements of two 'input' points, proportional to two numbers X and Y, force the 'output' point to move a distance proportional to the product XY. The 'output' movement of one unit is used to drive an 'input' of another by connecting them together with steel tape. In this way it is possible to build up analogues of quite complicated mathematical operations. Furthermore, when the units are suitably interconnected in *closed* loops, the resulting assembly is an automatic equation solver or computer.

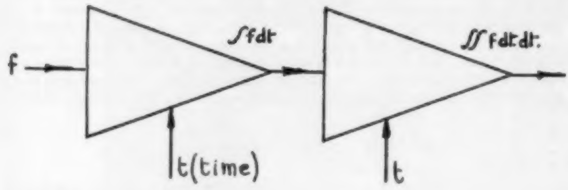
Computers, both mechanical and electronic, are playing an increasing part in all branches of engineering and it is important that they should be included in an engineering teaching syllabus. The computer here described can be used to provide an introduction to the basic principles of analogue machine methods in addition to providing physical systems which exemplify basic mathematical operations and equations. The latter aspect is believed to be important in helping the student with only average mathematical ability to appreciate the physical applications of mathematical equations.

Applications in Technical and Scientific Education.

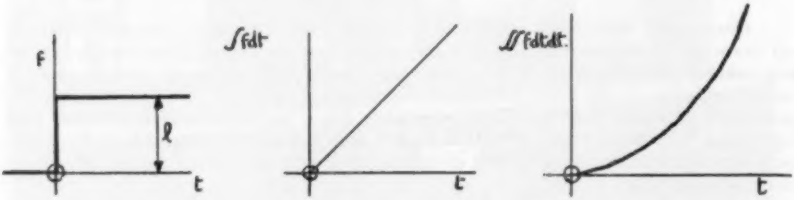
Although many engineering and physics students acquire a reasonable facility in solving mathematical problems, fewer of them have the ability readily to specify a physical problem in mathematical form or finally to interpret the results of the mathematical analysis in physical terms. Analogue computers (far more than calculating machines or digital computers) are ideal tools to use in training students to make full use of their mathematical knowledge. This is because analogue computers are working physical systems governed by mathematical equations in an obvious and direct manner. Students enjoy using analogue devices and this enhances their educational value. The value of the Air Trainers Link computer will be illustrated by two examples, fully discussed below.

Example 1.

Let two integrators be connected in tandem so that the output of the second is the integral of the output of the first (both integrations are with respect to time).



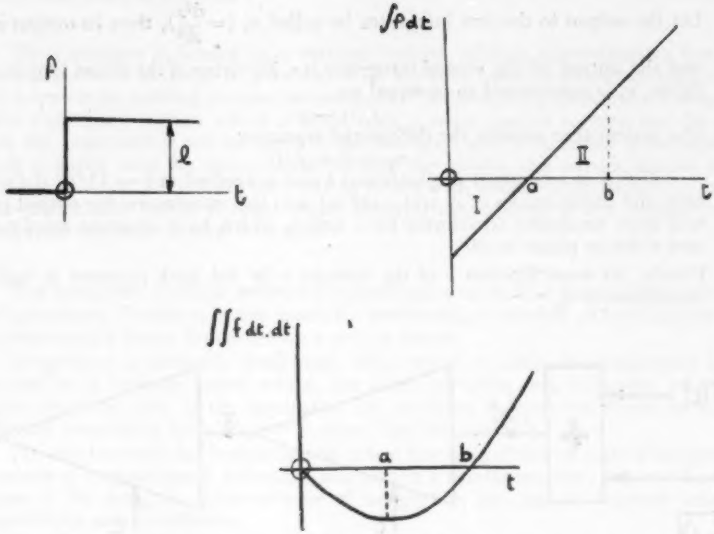
(i) Let f, the input to the first integrator be a constant displacement l applied suddenly at $t = O$. The output displacement of the first integrator increases at a constant rate proportional to l and the output of the second increases with constant acceleration:—



(ii) The results shown in the diagrams are obtained when the initial conditions are such that the output displacements of both integrators are zero at $t = O$.

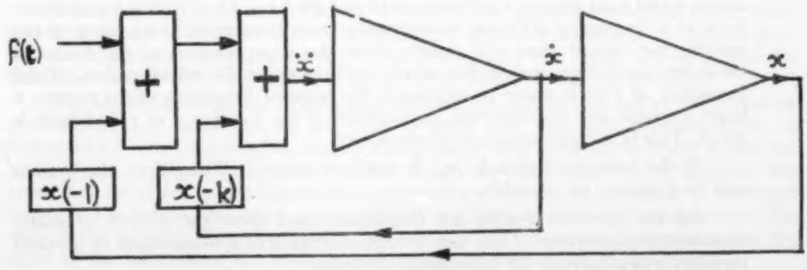
It is now apparent that 'initial conditions' in mathematical problems are set into the components in an obvious and direct manner. The principle of integration is exhibited visually in that the rate of change of the output is proportional to the input signal and indeed all analogue computers work in this way.

If the initial output of the first integrator is set negative, then the results will be:—



The output of the second integrator reverses at '*a*' when the output of the first passes through zero, and it becomes zero at the time '*b*' when the areas marked I and II are equal. The output pens of the computer actually draw these graphs.

(iii) Next suppose that the output of the second integrator is reversed in sign, fed-back to an adder, there added to $f(t)$ and the sum fed into the first integrator.



Let the output to the first integrator be called x_2 $(= \frac{d^2x}{dt^2})$, then its output is x_1 and the output of the second integrator is x. By virtue of the closed loop in the figure, x_2 is constrained to be equal to:

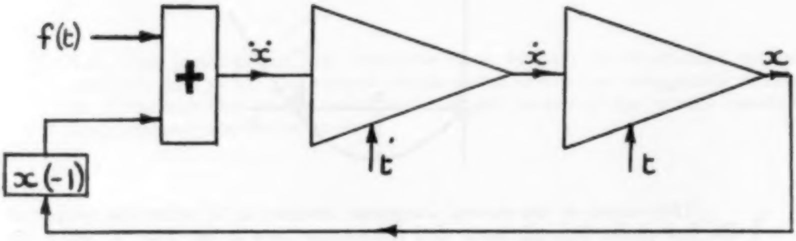
$$f(t) - x$$

The system thus satisfies the differential equation

$$x_2 + x = f(t)$$

If $f(t)$ is a constant displacement l and is applied at $t = O$, or if $f(t)$ is zero and initial values of x_1 and x are set into the integrators, the output pens will draw sinusoidal waveforms for x and x_1 which have constant amplitudes and differ in phase by 90°.

(iv) Finally, let some fraction k of the velocity x be fed back reversed in sign to contribute to x_2,



then $x_2 = f(t) - kx_1 - x$
or $x_2 + kx_1 + x = f(t)$
which is the differential equation of a simple damped vibration.

If initial values of x and x_1 are set in and $f(t)$ is a constant applied at $t = O$, the result is a damped oscillation of such magnitude and phase as will retain x and x_1 at their initial values until just after $t = O$. If $f(t)$ is a sine waveform of a frequency differing considerably from the natural frequency of the system, the output pens will clearly show the superposition of the transient decaying oscillation and of the steady non-decaying forced vibration. If the frequency of $f(t)$ is made to approach the natural frequency of the system, a large response will be observed, particularly if the fraction k of x_1 fed-back is small. This is the phenomenon of resonance.

If the velocity-feedback (x_1) is positive instead of negative, the system will be found to be unstable.

All the previous results are important, and form the subject of many examination questions! They can be demonstrated to a whole class or worked through quantitatively by individual students.

TECHNICAL DESCRIPTION

General

The computer is housed in a vertical wooden cabinet approximately five feet square and two feet deep. It is mounted on a plinth twenty-one inches high fitted with a drawer for holding accessories and tools. The computing elements are mounted in the right-hand section which is fitted with a roller shutter to keep out the dust when the computer is not in use. On the left-hand side is mounted a recorder on which a paper strip six inches wide records the results and carries special input functions. Also on the left are four setting knobs with scales for setting in the values of co-efficients. A switch is fitted which controls the running of the computer and enables the operator to stop it at will.

Scope

The computer contains facilities for performing up to four integrations and four multiplications. Provision is also made for performing a number of summations and for converting a linear function into a sine or cosine.

Integration is normally performed with respect to time, the integrators being operated by a constant speed motor, but other variables can, of course, be scaled against the time scale of the apparatus. No provision is, however, made for simultaneously integrating with respect to more than one variable.

Provision is made for feeding in one or two functions of the variable of integration by means of manual graph following, and also in a similar manner, any one function of one of the integrals. The variable of integration can also be injected into the computation as a co-efficient.

A wide range of scaling is available within the computer, so that integrations against time can be scaled down as required for demonstration and study. Computation can be stopped at any time so that instantaneous values of variables can be studied.

The input-output recorder

To facilitate subsequent analysis of results and to provide automatic correlation of inputs and outputs, they are all drawn on one strip of paper, about six inches wide, whose lengthwise scale represents the independent variable. The paper is automatically traversed lengthwise, at a rate proportional to the variable, past the recording pens and graph following pointers.

The recording pens are attached to two of the integrators which move them across the paper so that the linear displacement of each transversely to the independent variable axis represents the appropriate function. The curve drawn is thus the Cartesian relationship between the independent variable and the function.

The input functions are drawn as graphs on the paper strip before computation commences, and these graphs are then brought into line with pointers fitted to sliders which can be moved manually along guide rails fixed transversely across the track of the paper strip. Attached to the sliders are steel tapes which transmit the motion of the slider to the appropriate computing element. As the paper is traversed under the pointers, the latter are moved as required so that they remain over the line of the graph. Thus the linear displacement of the metal tape represents the value of the function of the independent variable represented by the graph.

Constant co-efficients

A group of four circular knobs fitted with pointers and scales is mounted beside the recorder. These knobs are connected by steel tapes to appropriate parts of the computer so that rotation causes linear displacement of the tape and of the computing element to which it is attached.

The computer

The computer is built up of a number of standardised computing units, multipliers, integrators, summers, etc., which can be readily interconnected in a wide variety of patterns to represent analogues of physical phenomena or differential equations. Provision is made for scaling the interconnections appropriately by means of variable ratio levers in addition to the four multipliers. These variable ratio levers are set before the computation commences and cannot be changed during the computation.

Signals, in the form of linear displacements, are transmitted between the units by means of stainless steel tape which is maintained in a state of tension throughout. Thus hysteresis or lost motion is virtually eliminated. The resolving power of the computer is better than one part in three thousand.

The following equations will give some idea of the possibilities of the machine.

$D^3\theta + aD^2\theta + bD\theta + c\theta = f(t)$

$D^2\theta + aD\theta + bg(\theta) = f(t)$

$y_1 = ay + bx;\ z_1 = cy + dz.$

$\int f(t)\sin at\ dt$

Generation of $a + bt + ct^2 + dt^3 + et^4$

These are merely a few of the wide range which the machine can handle.

FOUR POINT CONICS—A MATHEMATICAL FILM

A new mathematical film by T. J. Fletcher has just made its appearance under the title 'Four-Point Conics.' It displays many of the properties of pencils of conics and of the eleven-point conic which are proved in the standard textbooks on projective geometry. We hope to publish a review of the film in our next issue. The film is in black and white on 16mm stock and may be purchased, price £10 per copy, or hired for 7s. per day. Running time is 13 min. Enquiries concerning the film should be sent to Mr. Fletcher at Sir John Cass College, Jewry Street, London, E.C.3.

SECONDARY MODERN SCHOOL MATHEMATICS—IV

BEGINNINGS OF ALGEBRA

J. V. TRIVETT

The word *algebra* is one of those which in the educational vocabulary suggests to many people a ceiling above which they never rose, or at least a topic which they did not conquer, and which since their school days they have abandoned as having no relevance to their lives. To teachers it is too often an impossibility for the pupils either by age or by virtue of that mysterious quality 'intelligence'. Can we say why this attitude is prevalent and whether it is inevitable?

In a well-known, much used series of Algebra textbooks, this is said: '. . . the usual difficulty for the boy beginning algebra is in the transition from numbers to letters.'

This provides a valuable clue, for in fact there cannot be *any* transition from numbers to letters. Numbers are abstractions, relations, things of the mind; letters are marks on paper which have meanings outside themselves. The marks used in arithmetic are not numbers, they are figures or numerals, and if a transition from figures to letters is meant, then again a misleading attitude is perpetuated. Let us examine the reality a little closer.

The word 'dog' shown to a non-English speaking reader will not convey that the ink-symbol represents an animal with a particular shape, colour and use. The reader has to *learn* that 'dog' is related to the animal in his experience; only perhaps if the word is an onomatopeia is any clue automatically given to its meaning. Likewise the mark '6' gives no indication of its meaning to one who cannot read the language of Arabic numerals. It is necessary to learn that '6', as well as 'six', represents something which is not there on the page of the book, that it is a symbol. It is also impossible for anyone to sit and contemplate the writing '$6 + 2$' for the first time and not knowing what it means come up with the answer '8', although progress might be made with / / / / / / and / /. Of course, it is fairly easy to reach a position by coaxing, reward or punishment, in which when a teacher says 'six add two', this sounding like the bell for food to the Pavlov dog, then the pupil immediately says 'eight'. But the difficulty in this lies in the fact that it works well enough when a similar situation is repeated, but when the stimulus goes or changes as it inevitably will when the child leaves school, and the situation looks new or is strange, then the same facility for correct answers will not necessarily be available.

Algebra is not then an introduction suddenly of different symbols, letters perhaps, to represent familiar symbols, say figures. Nor is it concerned primarily with using symbols for numbers for we now see that that has always been done since the child first wrote a numeral standing for a number. He has of course, been taught by habit to accept that he understands $6 + 2 = 8$ when he can repeat the reading of this writing, but careful thought will show that the reading and the writing are only so by common consent in the western world. The underlying truth, however, is universal and does not depend on the conventions.

Rather is algebra concerned with what can be *done* with numbers, and so long as this is the emphasis it is not important how it is recorded on paper, though mostly we shall still use conventional symbols—after all it saves trouble and conveys information easily to others. Seen thus algebra can begin in the infants' school, continue throughout junior school and no bifurcation of the subject is necessarily suggested by having secondary modern and secondary grammar schools. For, to take a simple example, even children of six or seven years can appreciate that the commutative rule is true when applied to the addition of whole numbers, i.e. $3 + 2 = 2 + 3$ and similarly for *all* other pairs. If, however, the teacher considers it satisfactory if the pupil merely knows that '2 add 3 is 5' and is brought up solely on a diet of right answers, the task is made more difficult for the later introduction of algebra, and the child meanwhile misses much for which it is ready. The algebra in this example is the awareness that regardless of what two numbers are used the order of addition is irrelevant. Mastery of this can become an integral part of the children's 'mental tools'. It is, after all, a very practical matter indeed to know how best to add a column of figures. Surely then, we want everyone who has to do this to think *for himself* of an economic way of so grouping the figures, forming the subtotals, in order to obviate mistakes. Yet one will hardly do this spontaneously unless the habit is engendered based on an appreciation of the reasons.

Such 'dynamic' understanding of the 'grammar' behind the conventional answers can permeate every lesson in mathematics and we can list the incidence even in conventional arithmetical situations:

(i) All four basic rules are interpretations based upon *addition*. If it is known say that $2 + 2 + 2 + 8 = 14$ we may ask:

(a) 'How many 2's added to 8 give 14?' We write $3 \times 2 + 8 = 14$ and we have awareness of the operation called *multiplication*.

(b) 'What added to $2 + 8$ gives 14?' The answer of $2 + 2$ can be written $14 - (2 + 8) = 2 + 2$, and we have *subtraction*.

(c) 'What number added three times gives 14 subtract 8?' This gives us *division*, writing $(14 - 8) \div 3 = 2$.

Thus addition and subtraction constitute a pair of *inverse operations;* so do multiplication and division. And they are all basically *additions*.

(ii) 37 represents 'thirty-seven' only in 'scale of ten'; i.e. 37 means '3 tens and 7 ones'. In 'scale of eleven', thirty-seven would be written 34 (3 elevens and 4 ones), 41 in scale of nine, etc.

If we can work in any scale we have eliminated all difficulties with weights and measures sums, provided only that we learn or are told which scale is appropriate.

e.g.	37	This is correct in scale of 12, which is appro-	3s. 7d.
	+26	priate in situations of shillings and pence,	2s. 6d.
	61	so . . .	6s. 1d.
	37	scale of 8, e.g. gallons and pints . . .	3 gall. 7 pt.
	+26		2 gall. 6 pt.
	65		6 gall. 5 pt.

37 +26 — 513	scale of 20, e.g. tons and cwt.	3 tons 7 cwt. 2 tons 6 cwt. ——— 5 tons 13 cwt.

(iii) 93×27 — the answer is 2511 but there are many ways of getting it. Even conventionally

$$93 \times 27 = (93 \times 20) + (93 \times 7)$$
$$= (90 \times 20) + (3 \times 20) + (90 \times 7) + (3 \times 7)$$

Each of these four products can be worked in the alternative order and the four can be totalled in any of the twenty-four ways!

This is true for the multiplication of any two numbers each represented in binomial form (i.e. two terms connected by + or —). By all means have children learn that they can economise on paper by writing two lines before adding, so long as they can remember the 'carrying' figures. But if the criterion of success in multiplication is to be the use of as few written steps as possible, then the second of the writings below although in conventional form is bettered by the third example. In this *all* 'carrying numbers' are retained mentally and only the final product is written. The *quickest* way of course is to know the answer immediately, previously learnt by heart!

$$\begin{array}{r} 93 \\ \times 27 \\ \hline 21 \\ 630 \\ 60 \\ 1800 \\ \hline 2511 \end{array} \qquad \begin{array}{r} 93 \\ \times 27 \\ \hline 651 \\ 1860 \\ \hline 2511 \end{array} \qquad \begin{array}{r} 93 \\ \times 27 \\ \hline 2511 \end{array}$$

In these examples the algebra of the situations is stressed. The 'answers' are there and are not to be ignored but it is the behaviour of the operations which is really important. All the time we are trying to emphasise how the numbers behave, what are the structures, and what can be done with certain operations, what can not. The outcome is to be mathematical *thought* ready for application to real-life situations instead of the usual hodge-podge of unrelated facts seldom to be seen as solutions of practical problems, no matter how correct the facts may be.

What has been suggested can and should come in the primary school years but if it has not it must come at the beginning of secondary school life if any real progress is to be made in mathematics. In this case—and this will probably be the case for some years to come—fresh approaches are vital for we want to avoid as far as possible any suggestion to the pupils that they must begin again to enrich their knowledge and cast off their prejudices. Here, therefore, are some suggestions for invigorating each of the topics listed above when we want the algebraic attitude highlighted:

(1) Write the numerals from 1 to 100 in ten rows, 1 to 10, 11 to 20, etc. Operate within this table by picking a number; then move a finger or an eye to another number by one or several straight moves. Thus 28 $\rightarrow$ = 29, the arrow denoting one move from 28 to the right. Similarly, 28 $\rightarrow \uparrow \nearrow$ = 10. What is the inverse of $\rightarrow$? What two operations therefore give the equivalent of an 'identity operation', i.e. they are equivalent to no operation at all, leaving the number identical? Clearly $\rightarrow$ followed by $\leftarrow$ does this.

What four operations, five, six . . . are equivalent to an identity operation?

If instead of this the operation 'add' is considered, and numbers are used instead of numerals, what now is the identity operation? In other words, what number added to x, say, gives x? What two-step equivalents are there? (+ 2 — 2, + 7 — 7, + y — y . . .); four-step equivalents and so on?

What if the operation is multiply?

(2) The scale of 2 is the *binary scale* and is used extensively in electronic computers. Instead of one, tens, hundreds . . . columns, we have ones, twos, fours, eights . . . Thus, instead of 0, 1, 2, 3, 4, 5 . . . we get 0, 1, 10, 11, 100, 101, . . .

The addition and multiplication tables become

+	0	1
0	0	1
1	1	10

×	0	1
0	0	0
1	0	1

Sums may now be worked in binary notation.

The writings in different scales may be read from the 0 – 100 table:

	0	1	2	3	4	. . . . .
Scale 2	10	11	12	13	14	. . . . .
Scale 3	20	21	22	23	24	. . . . .
Scale 4	30	31	32	33	34	. . . . .

The first four numbers in scale of 2 are in the smallest square; the first nine numbers in the scale of 3 are in the next square; and so on.

(3) Introduce the words *monomial*, *binomial*, *trinomial*, etc. Write numbers in these ways:

	Five	'a'	'x + y'
Monomial	5	a	(x + y)
Binomial	3 + 2	a + 0	x + y

Trinomial	$4 + \frac{1}{4} + \frac{3}{4}$	$\frac{a}{3} + \frac{a}{3} + \frac{a}{3}$	$x + y - 0$
Polynomial	$10-2+3-4-2$	$6a-0+4a-8a-a$	$3x-2x+\frac{y}{3}+\frac{y}{2}+\frac{y}{6}$

Begin with, say, $6 \times 8 = 48$, a product of two binomials giving a monomial. Write the six as a binomial:

$$(4 + 2) \times 8 = (4 \times 8) + (2 \times 8)$$

Write both as binomials:

$$\begin{aligned}(4 + 2) \times (5 + 3) &= (4 + 2) \times 5 + (4 + 2) \times 3 \\ &= (4 \times 5) + (2 \times 5) + (4 \times 3) + (2 \times 3),\end{aligned}$$ a quadrinomial,

. . . . and so on to

$$(n\text{–omial}) \times (m\text{–omial}) = (n + m)\text{–omial}.$$

(4) Do some work on measuring areas of *irregular* shapes using triangles, circles, rhombi as the units. Discuss the edge problem when the units do not cover the area an integral number of times.

Do the units have to be regular?

What is the rule for calculating the area of a triangle, square, etc., when the unit of area is an equilateral triangle?

Volumes can be dealt with by using cubes, cuboid shaped bricks, ping-pong balls, etc.

Pupils can explore independently the relation between perimeters, areas and volumes. Using perhaps loops of string different shapes can be traced onto paper, all having the same perimeter lengths; must they also have equal areas? Cut a rectangular sheet of paper and arrange the pieces to give another shape. Has the area changed? Has the perimeter?

A NEW APPROACH TO THE TEACHING OF ALGEBRA

OLIVE TOLSON

At a meeting of the West Riding branch in May, 1958, it was decided to form sub-groups, one for each type of school, to promote discussion and the preparation of teaching experiments and investigations. In consequence a small group of teachers from four selective schools met and discussed the idea of a new approach to Algebra. In the past the usual approach has been through generalised arithmetic and equations of the "think of a number" type; the new idea was to begin with literal equations—as far as we knew this had not been attempted before.

After much discussion we planned a scheme of work and each member undertook to write the preliminary sets of notes for one section of it. We borrowed Cuisenaire Rods from various sources, each had a complete set of notes and we were ready to begin our experiment with the first year entrants in the autumn of 1958.

The pupils were given the rods, allowed to handle them and see that all the rods of the same size were the same colour, and it did not take long to get them to suggest instead of talking about the black rod, the red rod, and the white rod, that it would be quicker and easier to denote them by B, R, and W. As some of the initial letters of the colours were the same, the teachers had evolved their own code "N" for blue (navy), "H" for brown (Don't just say brown . . .), "D" for dark green, and "L" for light green. However some of the classes did not agree with these and were allowed to use their own code letters "S" for sapphire blue, and "E" for emerald green, etc. The next step was to see that a yellow rod and a red rod placed end to end were equivalent to one black rod. This was expressed in the form $Y + R = B$. By actually handling the rods and arranging them on the desks the pupils found that the same three rods gave rise to the expression of the same fact in four ways:— $Y + R = B$; $B = Y + R$; $B = Y + R$; $R + Y = B$. This was followed by establishing the idea of subtraction and arriving at $Y = B - R$, etc. After practice with the rods the pupils were given exercises of the same type but using different symbols.

We imposed the rule that only two colours could be used and this led to equations of the type $3R = D \therefore R = \frac{D}{3}$. More colours were allowed leading to equations such as $L + R + C = N$, and the introduction of brackets when changing the subject. $L = N - (R + C)$ or $L = N - R - C$.

In February, Mr. Collins set a test paper, which was taken by all classes using the rods. The test was in three parts, twenty minutes being allowed for each. The first asked for illustrations of given equations; the second was on changing the subject of equations using only the letters associated with the rods, but the pupils did not have the rods to handle; the third part had equations using the other letters and symbols. One or two questions were given which led to a numerical value for 'x' and it was interesting to note that many pupils performed the operations correctly, but did not work out the answer e.g.

$$8 - x = 2 \therefore x = 8 - 2$$
$$3x - 4 = 8 \therefore x = \frac{8 + 4}{3}$$

This was only to be expected as we had been attempting to teach operations and symbols, not how to find an unknown number.

The group met once or twice each term to discuss progress and problems which had arisen. In July we planned the revision of the notes. These have now been published and copies may be obtained from "Mathematical Pie", or from the West Riding branch secretary, price 2/-d.

The group would be pleased if other teachers would try out the scheme by following the notes and let us have comments on the results of their experiments; we feel that the method has much to recommend it; particularly for the not-so-bright child. It has been most stimulating for the teachers, one of whom found herself teaching permutations to her first form girls as a direct consequence of using this method.

BLACKPOOL CONFERENCE 1960

"EDUCATING THE EDUCATOR"

Here we were again; this time would-be educators who would be educated. The theatre was the same; the play was part-Restoration, part 20th century; the producer was the talented, now literary mystic, Mr. C. Birtwistle of Nelson; the cast of stars included (no seniority in the billing) Mr. R. H. Collins of Doncaster, Mr. J. V. Trivett of Bristol, and Mr. D. H. Wheeler of Leicester. With acknowledgements to Continuity by Clutten, Harmony by Harris, Balances by Briggs and Refreshments by Redmans, the stage was set for yet another production by the A.T.A.M. Travelling Players.

Opening the course, Mr. Trivett informed us that our President, Dr. C. Gattegno, sent us his greetings. A warm welcome was given to Mr. Collins after his recent illness, and he did not look to be really fit as yet. Mr. Trivett felt that problems in the teaching of mathematics faced us all. His introduction of the main speakers for the course excluded lettered qualifications for, he felt, degrees were not certificates of wonderful wizardry and we must be prepared for some re-education by the pupils.

The Mathematical Education of the Teacher

Mr. Birtwistle opened his talk on this subject by saying that teachers attending courses such as the present one were sometimes disappointed by the fact that we did not have a large display of teaching aids. At some of our meetings we did have such a display, but at this conference we were going to consider the most effective use of the most important teaching aid of all—the teacher himself. The mathematical ideas and modes of thought which may be wrought into our minds by a suitable mathematics course are an asset of permanent cultural value, but we must remember that it is the ideas and modes of thought which form the element of a liberal education and not the manipulative dexterity that goes with the mere solution of problems. Once we laid emphasis on techniques we tended to avoid the possibility of a liberalising influence.

Properties and methods of mathematics had their uses and importance in a world where the affairs of life were becoming increasingly amenable to mathematical treatment, but to study these alone was not the end to be sought. It is for the mathematician to attain to an understanding of the *thought* behind the thought of the subject—the essential philosophy that is part of true education. If we are to be effective as teachers of mathematics it is not sufficient to know the small region of mathematics which we are to teach in our daily work; one has to know what mathematics is, what it is about, what it is capable of doing, and the most effective way to use it. The mathematics we were teaching daily would then be put into perspective. Hence our mathematical education was never-ending. It was in this light, Mr. Birtwistle concluded, that he hoped we should approach our work for the weekend.

Mathematics — The New Point of View

The self-styled Public Relations Officer for the New Mathematics, our friend Mr. Hope, declared himself the herald of New Mathematics, partly self-appointed. He apologized for being neither a mathematician nor a teacher, but just a lecturer in a training college. As Shaw says: those who can, do; those who can't, teach; and extending this, those who can't teach go into a training college and teach others to teach!

We had it affirmed that there had been a mathematical revolution in the last fifteen years. In 1959, out of a 600,000 age group, only 400 took degrees in mathematics. All the hard work is being taken out of industry but in its place we need workers with an increased ability in mathematics. There could arise the situation where we find factories full of wonderful machines without men and women with enough mathematical ability to use them. This is the challenge! We must produce mathematicians of various stages of ability to fill the forthcoming gaps in industry.

Mr. Hope pressed the point that we were now looking for an algebraic base for geometry. He said that it has been suggested that an algebraic axiomatic way of teaching geometry is impossible, but in the world of pure mathematics work was being done along these lines. We were told that mathematics is now respectable and were advised to look on the back pages of *The Times* or *The Observer*. Even the Lyons' catering firm used a computer to work out how many pies had to be baked after telephone calls had been sent in each evening. Mathematics here would look for a method which could be justified logically so as to be fed into the computer.

The approach to algebra should strengthen work already being done in enlightened infants' schools. Instead of 'mucking about' with an undefined set of elements at 13, we should be using a defined set of elements at 7. In geometry, we were informed, physics should be the base. Much work in defining the relation of equivalents could be done and the children could develop naturally from the concrete to the abstract. In work on congruence the axioms of reflection can be used and the children can see that a point is reflected as a point and a line is reflected as a line. The approach through surveying should be made, and much work can be done with vectors. Geometry as it is often taught finishes at G.C.E. 'O' level; mechanics and statistics don't need it, and all that riders do is to show how wonderful congruence is.

Mr. Hope closed his talk with specimens of linear programming, outlining a fictitious foodstuffs packaging problem.

This was the first evening then. Birtwistle and Hope setting the pace and the background for the weekend which now might be re-titled as *Mathematics has come to stay!*

Group talks

Saturday morning found us wending our way to various groups to consider topics for future study. Opening his talk on *Stage 'A' Calculus*, Mr. Hope said that we were to consider the preparatory stage and must consider why we should teach the calculus at all. If we made a few simple headings we could see where we were going, and could use basic algebra in a possibly more interesting way. We would find the idea of Limit, Slope (Rate of Change), Maximum and Minimum, Series, and Area of Irregular Figures in our headings.

Mr. Hope gave as one example of limit the finding of the circumference of a circle of known radius. This was the method of exhaustion where we imagine the circle to be circumscribed by a polygon of any number of sides n and to have a polygon of the same number of sides inscribed in the circle. Having C as the perimeter of the circumscribing polygon and I as the perimeter of the inscribed polygon, we have C always greater than, and I always less than the circumference of the circle. $C - I$ becomes less and less as n is increased.

The idea of slope as rate of change should come from the use of straight lines in a graph, the slope being the same at all points on the line because it always makes the same angle with the X-axis. Having found that a straight line has a constant gradient, we can say that the rate of change of y with x is constant; but as most quantities change at a varying rate, we find that their graph is a curve. This latter can be clearly seen when, for the same increases of x, different curves show different increases in y (rate of decrease to be regarded as negative rate of increase).

On we went with Mr. Hope, seeing many cute and acute illustrations of the development of calculus, suitable for most children and including many dynamic applications. We had the idea of rate of change shown by having a flywheel toy car moving down a graduated slope of, say, six feet, and stop-watch timing taking place (say the mean of three). The scales could be drawn on graph paper, highly magnified, and the average speed at various times could be calculated.

The application of differentiation to maximum and minimum brought in the ideas of maximum volume of a box, least surface area for constant volume (as in a cylinder), maximum volume of a sphere, etc. Having been given *Fundamental Mathematics* by Richardson as a book for future study, we adjourned for coffee.

An Approach to Statistics. In a precise and illuminating manner, Mr. J. V. Trivett showed how simple a subject statistics is to introduce to children. To make the subject interesting and understandable, all data should be collected by the children from within the child's own experience. This is then organized by them and graphs drawn after much class discussion.

Examples of subjects suggested were: number of petals of a daisy; weight and height of children at school; direction of cricket balls during a match; classification of articles in a newspaper; the most used letter in the English language; etc.

Mr. Wheeler gave a most illuminating talk on the *Algebra of Sets.* He approached it as a teacher giving an introductory lecture to students who had not before met the subject—this was most appreciated. Using members of his audience for his sets, he established the fact that with two criteria to be observed there were four sub-sets; with three, eight sub-sets. From thence he proceeded to explain the notation used, in particular union and intersection, and in spite of blackboard difficulties gave a clear explanation. He demonstrated his points geometrically at this stage. He also gave a practical classroom example mentioning the use of sets to find H.C.F. and L.C.M.

He then introduced the nul-set and the universal set and showed how the operators $\cup$ and $\cap$ applied to these sets, drawing an analogy between O and 1 and the operators $+$ and $\times$. He left his audience with what he described as a 'toe-hold' in the subject, and an appetite to learn more.

Mr. Collins opened his talk on *Other Geometries* with a challenge to the place held by Euclidean geometry and its descendants. Euclid, he quoted, was a tactile geometry. The phenomenon of perspective makes it quite clear that one does not see parallel lines as lines which never meet; on the other hand it is easy to perceive them close at hand. It becomes a matter of investigation how other axiomatic systems can help in the presentation of the idea of proof and in the solving of problems in the real world. It was necessary to distinguish clearly between the axiomatic system on the one hand and the applied mathematics which is the use to which a geometry is put. We were asked to consider if a talking fish could describe wetness, or what a woodworm could know of two-dimensional space, let alone three-dimensional space. Thus just as the superposition of the mirror image congruent triangle requires a move from two into three dimensions, so the physical identification of left and right hand gloves of a pair would invite a visit into a fourth dimension.

The relativity of definition was another point. As for Humpty Dumpty, words mean what we say they mean, no more and no less. Some members of the group clearly felt a difficulty here, and it was suggested that the inevitable lack of a basic referent introduces a difficulty in discussion. Thus we fall back on saying words and then follow them up with "But that does not mean what it appears to mean; it is just a series of sounds." This was how Mr. Collins introduced a miniature geometry demonstrating that a very few axioms could lead in a very simple way to a meaningful proof system and also to practical uses. He used the words 'points' and 'lines' and even a diagram which had points and lines, to demonstrate his geometry uttering as he did so the incantation quoted above. He then used his system in the applied mathematical problem of the random arrangement of plots in an agricultural experiment. He ended with a plea that investigations by teachers of the other geometries that have been invented, though not easy, will provide illuminating and valuable mathematics for use in school.

Communicating our knowledge

Mr. Hope, addressing the whole gathering on this topic, quickly used one of his Hope-isms—one I have heard him use a time or two for, and with, effect—"Every child must be given the right to be wrong!"

He had us working immediately! We were given the takings of two shops for a period of eight weeks, those of Shop I being in round hundreds, and Shop II having multiples of 25. We were then asked which was the better shop, advised to look for a pattern, and told that all answers could probably be justified. We should use numbers which are relevant and should not set examples so that we have time to get on with our 'pools', remarked Mr. Hope. "Simplicity breeds confidence; confidence begets courage. We need courage and conviction," he said.

Given 36 one-inch squares, to form all the rectangles which can be made is a good exercise, it was suggested. If the rectangles were cut out in paper and pasted on a piece of card, valuable discussion on the perimeter could be made. What sort of rectangles can we make?

$$\begin{aligned} P &= 36 \\ b &= 1, 2, 3, \ldots\ldots \text{ etc.} \\ l &= 17, 16, 15, \ldots\ldots \text{ etc.} \\ \mathrm{A} &= 17, 32, 45, \ldots\ldots \text{ etc.} \end{aligned}$$

All belong to the same family.

Graphing of taxi mileage, parcel post, letter rate, etc.; factorization practically, by using strips of card, and on to the difference of two squares; the idea of a limit; all were illustrated. We were told that arithmetic, algebra, geometry and co-ordinate geometry could and should be integrated in the early stages. The final advice was: "Find a pattern through the mathematics scheme which fits the particular school, the particular children and yourself."

The underlying principles of mathematics

Mr. Wheeler felt that a suitable sub-title would be 'The underlying principles of teaching and learning methods'. Methods cannot be divorced from the people who use them, and there is an affinity between the people who are learning and the people who are transmitting mathematics. All people, of all ages and ability, learn better from concrete rather than abstract experience. Some people think that it is a good thing for Secondary Modern Schools but that it would get in the way of mathematics in a Grammar School. This assumption has its dangers. The dangers of the concrete approach are such that we force things. Mr. Wheeler told us of a student who told the class that the sum of the internal angles of a triangle was 180°, then told the class to 'experiment' and 'find out for themselves'.

Some comment was aroused in the room when we were told, "The use of measuring instruments to prove geometrical propositions is invalid". An expansion of this statement was asked for and Mr. Wheeler told us that, in his opinion, mathematical experiment was not of the same kind of order as in physics. Measurement could obscure what we were trying to show. The type of experiment was not as in science. There need not be several measurements as we often have no feeling of wanting to do a thing more than once for confirmation. There is a level of verification appropriate to the mental level of the child at all ages, but in later stages the result must be expressed in some way by some form of communication. Notation only comes after the desire to communicate and our responsibility is to introduce the right notation at the right time. We must avoid rigidity; for example, always drawing right-angled triangles with the right angle at one end of the base.

After questioning himself as to whether he had spoken about what he was supposed to speak about, Mr. Wheeler smilingly concluded, mentally making a gesture, I felt, in the general direction of any criticism.

Group talks

Our last session for Saturday was in groups to talk about practical difficulties in the classroom. Discussing *Constructing a Syllabus*, the group leader, Mr. Birtwistle, began by asking whether a syllabus should be a detailed one requiring rigid adherence, or whether it should be an outline which would be filled in by the teacher. The choice was felt to depend largely on the staff available; the former alternative ensured an integrated syllabus with any staff, but the latter was more desirable with a less enlightened staff. The practice of blindly following an examination syllabus or the topics listed in a textbook was deplored; the content of the syllabus must obviously be governed by any examination that is taken, but it is also a function of what we think ought to be contained, of the interests of the teachers concerned, and of the requirements of the children themselves. This last point led to a discussion of vocational work, and it was generally felt that this offered stimulus to the mathematics of the upper secondary school.

It was agreed that the lower streams should follow a separate syllabus and not merely a watered-down version of the A-stream syllabus, but the difficulty that then arose of pupils moving from stream to stream, or from set to set, was not resolved. Another important point was the place of revision; there was a danger of a class revising fractions so often that they did nothing else throughout the school! Revision should be embodied in a fresh topic, so that progress is continually made.

Types of syllabus were discussed and finally it was suggested that we have outgrown such things as involved multiplication (complicated fractions, simultaneous equations and commercial arithmetic), rigorous geometric proofs, and such long-winded items as the general method for square roots and H.C.F. and L.C.M. Mr. Birtwistle asked if these ought not to be replaced by the use of logs, the slide rule and calculating machines, trigonometry instead of formal geometry, and the basic ideas of the calculus. The ensuing discussion evolved various practical applications, statistics, football pools, hire purchase and income tax as topics suitable for this modern age.

Mr. Collins suggested many points for discussion on *The G.C.E.*, but most time was spent in discussing G.C.E. in Secondary Modern Schools. Some members of the group felt that to take G.C.E. exams in a school gave a certain amount of prestige to the school, both inside and out. Some progress had been made, especially in boys' schools, in having a chosen group taking G.C.E.

In both Modern and Grammar Schools there seems to be a difference between the attainment in general of the boys and the girls. It was suggested that the boys saw more use for their mathematics than did the girls and so made more effort and showed more interest. Some of this lack of interest and knowledge of basic principles was caused by lack of background and in some cases lack of knowledge in some Junior School teachers.

Text books, it was felt, were only needed to provide examples, and there was a difference of opinion about the use of tests given regularly. Some members felt they were of great value in assessing progress and assisting revision, but others thought that five minutes at the outset of a lesson was test enough. It was obvious that there were difficulties to be overcome in a G.C.E. course in all types of school, especially in teaching geometry, and further investigation was called for.

Mathematics and the Fine Arts

Sunday morning came and with it a mathematical piano recital by Mr. Birtwistle with this title. Claude 'Debussy' Birtwistle opened his repertoire with a few chromatic scales and proceeded to find some mathematics in them. Chord ratios was simple stuff, known even to the Pythagoreans, but we delved deeper to study the wave motion of the air particles set up by a musical instrument. Graphs appeared and we saw how loudness, pitch and quality all depended on mathematical characteristics, and a demonstration of Fourier series applied to the harmonics of a violin string using one of Frank Merry's ingenious straw-boards brought forth a spontaneous burst of applause from the audience; here was the perfect use of visual aids.

From the harmony of music to that of design, and we began to learn about lattices and tessellations as Mr. Birtwistle deserted the piano for a slide projector. We saw the results of an experiment with an art class of children to discover how far they relied upon mathematics in the creation of pattern, and then proceeded to Classical art. The influence of the proportions of the Golden Section was emphasized, and we were transported to Athens to study the mathematics of the Parthenon. Architecture led to sculpture, and what we thought to be examples of modern sculpture proved to be three-dimensional mathematical models based on equations in three unknowns!

Next came a quick tour of the art galleries and as we learned something of mathematical perspective and the influence of mathematical shapes in different styles of painting, we found that painters from Duccio to Picasso all owed something to mathematics.

But it appeared that this was not the end of our art lesson because cinematic art was also to be subjected to mathematical analysis. An amateur film maker, Stuart Wynn Jones, had made an award winning film *Raving Waving*, which was an attempt to couple abstract sound and movement. Wynn Jones, we were told, was not a mathematician, and it was interesting to see once again how the artist, this time in one of the most modern media, drew upon mathematics to achieve his effect. The film brought to a successful conclusion a talk which Mr. Birtwistle had described as 'an exercise in the use of visual aids'.

Group talks

The last group meetings were to discuss the wider aspects of mathematics. Mr. Hope, speaking on *Patterns in Mathematics* gave numerous examples of pattern in number sequences, in scientific experiments and in various branches of geometry. Pairs of sequences illustrated functional relationships which could be deduced and described by graphs. There was scope for research, particularly in geometry. For example, the conic sections could be rediscovered as the loci of intersections of two families of concentric circles, or of a family of concentric circles with parallel lines. Much work could also be done with new coordinate systems, with familiar loci emerging as functions of the coordinates.

The underlying principle was that of research. The pupil was confronted with a mass of information in which he saw a pattern, as a formula or a geometrical design, perhaps, from which he inferred certain theorems. The problem then was to find a method of proof, but the important thing was that the pupils should discover the patterns and evolve for themselves the laws relating them.

In *Mathematical Magic and Puzzles*, Mr. Birtwistle was up to his tricks again! This time card tricks, escape tricks, number puzzles and the like. The purpose? Why, he asked, does the popular press publish puzzles and problems for its readers? The answer is that these things are a challenge that few can resist; hence their popularity. By ignoring puzzles and tricks of a mathematical nature we were failing to make use of a valuable aid in our work.

The Mathematics of the Future (Mr. Trivett). In the discussion two experienced teachers gave a valuable commentary on the use of Cuisenaire material. Mr. Trivett enlarged on this as quite a number of the group had little or no experience of teaching with this aid. Nothing really new came out of the discussion.

The evaluation of our effectiveness

The final talk (to a much depleted audience, I am sorry to say) was given by Mr. Collins. We were asked to consider the internal written examination, presumably set on things done in class. Was not this a test of memory? What happens if someone else sets the test? What are our reactions? When the test is an external one and is set by someone else, the teacher often, realizing that this is again a test of memory, will look for the whims of the examiner or examination.

It would appear that there are at least two worlds in the teaching of mathematics. One world is to teach to pass an examination; the other is to teach mathematics as we think fit. One of the most unfortunate things that ever happened was to allow Secondary Modern School children to take external examinations. Examinations are assessing pupils, not the effectiveness of your teaching. If we, as teachers, accept the examination results as an assessment of our teaching, we can only accept them if we happen to agree with the aims and ideals of the examination. We must assess what we are trying to achieve in our lessons. We must search our souls when considering all these things.

What possible aims are there? Mr. Collins suggested the following:

1. Recording and memorising information.
2. Learning a knowledge.
3. Learning a technique.
4. Developing mathematical thinking.
5. Teaching to appreciate mathematics.
6. To apply mathematics.

Some of the aims definitely conflict with each other, admitted Mr. Collins. If we try to stick too readily to one we may be cutting our own throats, but there must be some degree of priority in our minds.

How can we look at ourselves and evaluate the quality of our results? We must remember that the ability to understand concepts is well-nigh impossible to test! There are so many points, but heading them all must be the virtue of self-criticism.

(i) Is the teacher the severest critic of himself the whole time? Is he prepared to 'slap himself down', if necessary, after he has given a lesson?

(ii) Has the teacher sympathy with the children?

(iii) Are we prepared to accept the child's proof?

(iv) Do we keep a record of courses of action which we have found successful?

(v) Can we pose situations and let the children do the talking?

(vi) Can we go into the poorer classes and get similar, comparable results? (A stony silence is the hall-mark of bad teaching)

(vii) Do we find time to study all the alleged incorrect answers?

(viii) Have we the frame of mind so as to be able to enter a classroom having a feeling of excitement, not of frustration?

These points, and others, were put so sincerely by Mr. Collins, that I had no doubt in my own mind that he attempted to practise what he preached.

And so the curtain came down, the final bows were taken, and the audience dispersed—some to have continued discussion, some to be replete, but all most grateful for the weekend's entertainment.

Report compiled and written by J. F. ELLIS.

(*The film 'Raving Waving' which was shown in Mr. Birtwistle's lecture is available on loan from the Grasshopper Group*, 33-35, *Endell Street*, *London W.C.*2.)

DON PEDRO PUIG ADAM

1900 – 1960

The man who bore that name died suddenly on January 13th last, leaving many grieved friends in many lands. He became known outside Spanish-speaking countries only in 1955 when he joined the International Commission for the Study and Improvement of the Teaching of Mathematics. In a very short time his delightful personality, his keen devotion to education, and his many talents gave him a place in people's hearts and spread his pedagogical influence beyond his usual province. Several of his writings since 1955 appeared almost simultaneously in Rome, Paris, Brussels, Madrid and even Warsaw, and his advice and sponsorship were sought by friends and journal writers. His death, at the peak of his productivity of thought, is a great loss for all, and we of A.T.A.M. who received inspiration from his *30 Lessons*, must express our deep regrets at the sad news of his passing.

Don Pedro, as he was called by all who cherished and respected him, was a member of the Academy of Sciences of Spain, Professor at the Faculty of Engineering of Madrid, in charge of the teaching of mathematics classes at Madrid University, and 'catedratico' at the S. Isidro Institute, a venerable institution going back to the Seventeenth Century. His writings for students include textbooks on *Metric Geometry*, and on *Differential Equations* that are authoritative and which greatly influenced higher education in Spain and Latin America (a place where he is well-known, but which he had never visited). But his main influence was at the Secondary level. Over thirty years ago he started writing textbooks which he constantly revised and improved, and which have renovated mathematics teaching in his country. Pupil and disciple of Don Julio Rey Pastor, the first real Spanish mathematician, he always acknowledged his debt to him by maintaining his name on texts which in 1924 were the outcome of discussions among them, but were looked at less and less by the master and clearly were only Don Pedro's own work. Many of the 'new ideas' now appearing in mathematical educational journals can be found in his several books.

In 1955 I met him in Madrid and we started thinking that the XIth International Seminar of the Commission could be held in Madrid in April of 1957. We planned it together, but Don Pedro took upon himself the very heavy burdens of being its local secretary. In our Bulletin, a longish report by T. J. Fletcher was a tribute to his courtesy, work and influence. The success of the seminar established in Spain a movement of reform in all directions, of which he was the indisputed leader. That meant adding much to his already heavy programme of work, and he 'burnt the candle at both ends', thus shortening his life by some years.

Don Pedro was a good realistic painter, whose portraits are both sensitive and sharp, produced by a loving person. He was a poet, although very few of his poems have so far appeared. His musical compositions were just now becoming popular and were being broadcast. But his most constant loves were his religion and mathematics. Always pious and tolerant, he remained an example of a faithful and charitable Catholic, observing his religious duties and knowing that God has His different ways for different people. His faith was his companion in the several occasions when he and his family met with dangers.

During his last year he produced logical machines which he patented. They are simple, ingenious and show the blending in one man of the engineer, the theoretician, the artist and the thinker. His wide readings, his encyclopaedic mind, his keen intelligence and alert sensitivity would make him grasp, long before many, what was of significance and of interest in large areas of human activity. His many friends in various fields vouch for his liberal interests and his ability to converse knowledgeably with specialists of so many fields. His literary command of Spanish and Catalan (this his mother tongue) enhanced his fluent and interesting speech. He read several languages and spoke some of them with ease.

In Spain he will live through the several generations of students he delighted and enlightened, and abroad, through his numerous devoted friends. As one of these who loved him most and received so much from him, I can speak on behalf of all those who are grieved. To his mother, wife, son and two daughters, to his two grandchildren, let me express here our Association's condolences.

C.G.

As one of the small group in our Association who knew Pedro Puig Adam I would like to add my tribute to his memory. I met him first at the International Commission meeting in Madrid, where a week's display of mathematical material from all over Europe offered teachers inspiration sufficient for a lifetime's work. Much of this exhibition was due to him, and it was an ideal setting in which to learn his very individual approach to our problems. At that time I did not know that he was in fact a musician, poet and painter, but we saw at once that he came to mathematics as all three.

For him teaching was a creative art, where the aim is to introduce the pupil to his environment, and where everyday objects provide themes for countless mathematical variations. Observe, reflect, interpret—in this way the concrete world around us is distilled into the abstract beauties of mathematics; which, in their turn, provide power to shape the future. Beyond all this, to his widely ranging talents there was added a gift for friendships that overcame the barriers of language.

Below there is a short extract from *El Material Didactico Matematico Actual*, the book which he wrote reporting the conference at Madrid. These modest paragraphs are in no way outstanding amongst his work, but they show the particular insight which he had, that power to create mathematics when others would have passed it by, which made him of all the teachers I have ever met the one from whom we could have learnt the most.

T.J.F.

THE GEOMETRY OF THE MUSIC STAND

PEDRO PUIG ADAM

Triangles, trihedral angles, various problems.

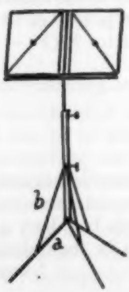
The legs of a music stand have the same geometrical structure as an umbrella (inverted) with three pairs of jointed rods. We can therefore obtain from it the same teaching situations as were explained in an earlier chapter "The umbrella, a multivalent model." (This showed how the study of an umbrella or more strictly a parasol, with straight ribs, provided twenty different theorems, problems or experimental situations for teaching geometry.—ED.).

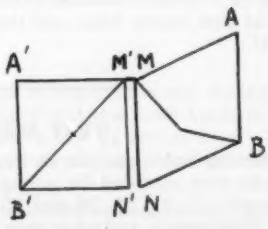
Spatial movements.

Displacement of the central rod along its axis gives the stand a translational movement in the same direction, rotational and helical movements are also possible.

Deformations which illustrate properties of the arc containing a given angle.

The stand itself is composed of two jointed rhombuses, with a common side. This double rhombus presents a variety of teaching situations. It is the same jointed



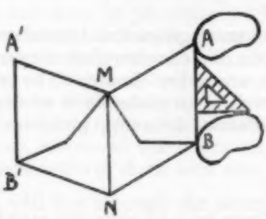


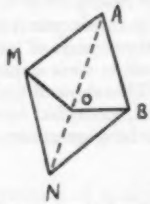
system which we encountered earlier in a lesson on inscribed angles and the arc containing a given angle. If the positions of the two sides NB, N'B' (or MA, M'A') are fixed the system has a single degree of freedom, supposing MN and M'N' to be coincident, and the angles B'MB and A'NA are invariant during the deformation and the point M (or N) describes the arc containing the angle B'MB (or A'NA) on BB' (or AA').

Plane translation.

If the side A'B' is fixed we have a jointed system with two degrees of freedom which allows the point A to describe any figure (within certain limits). Simultaneously the point B describes a congruent and similarly situated figure derived from the former by the translation AB.

Inversion.





Finally we consider the jointed system MOB which connects the vertices M and B of the rhombus; it is a system formed by two rods of length equal to half of the diagonal of the square formed by deformation of the rhombus. If we fix the point O in a plane the rhombus NBAM has two degrees of freedom which allow the point A to describe any arbitrary figure (within certain limits) and the figure traced by the opposite vertex N is the inverse figure with respect to the centre O, with power of inversion equal to $-\frac{1}{2}MA^2$; because OA.ON is the power of O with respect to the circle centre M and radius MA, and this is equal to $MO^2 - MA^2 = \frac{1}{2}MA^2 - MA^2 = -\frac{1}{2}MA^2$.

TEST SCORE

In a test containing four questions, six boys each answered exactly two questions correctly. No marks were awarded for wrong or incomplete answers. The scores of five of the boys were: 15, 21, 24, 26 and 35. The sixth boy had a different mark from any of these. What was it, and what were the marks for the individual questions?

(*Answer in next issue*).

MEASUREMENT IN THE JUNIOR SCHOOL

M. JOAN MEETHAM

Home-made measuring instruments

Many teachers of Junior children incorporate some practical work in their mathematics lessons. Sometimes, besides using prepared or commercial measures of various kinds, the children are led to make and calibrate their own instruments. This activity, as I hope to show later, deepens the children's awareness of several fundamental mathematical principles. But first, I should like to mention briefly some of the instruments which can be made.

A spring balance can be made out of a strong rubber band hung from a nail in a strip of wood—the position of the lowest point of the band when unstretched is marked on the wood as 'zero', and subsequent positions are marked with the values of the weights which are used to stretch the elastic—2 oz., 4 oz., 6 oz., etc. Better balances can of course be made with springs, though the children will need some time to experiment with springs first.

A force gauge can consist of a strong spring placed inside a tube closed at one end, pressed upon by a piece of dowelling rod which fits loosely in the tube and projects from the open end. The children can calibrate this by marking the position of the rod when various weights are placed upon it. This can be used to measure the force required to push open a door, or push in a drawer, etc.

Simple lever balances can be made for the children, and calibrated by them. For instance, one arm can be marked, in the correct positions, with the values of various weights which balance a given weight in a given position on the other arm. The steelyard can also be introduced, and it is possible for children, with help, to make one of these.

Time-measures of various kinds are rewarding to make, and of considerable historical interest. A water clock can be made out of two medicine bottles with a common cork which has a hole through it; and a sand clock similarly. A candle clock can be made by the calibration at regular intervals of the unlighted candle which is fellow to one being slowly burned down in a sheltered position.

Capacity measures also give plenty of opportunities for calibration practice—for instance, a child may graduate a bottle in tablespoonfuls, or a jam-jar in cubic inches.

Measuring out and measuring how much

It is sometimes helpful to a teacher to remind himself, during measuring work, that measuring can be of these two kinds, and that both kinds should be practised. Examples of 'measuring out' are 'Draw a line $2\frac{1}{2}$ inches long,' 'Weigh out 12 oz. sugar,' etc. Examples of 'measuring how much' are 'Measure this edge,' 'Find the area of this shape,' etc.

Calibration can be regarded as repeated measuring out, and since after calibrating an instrument a child naturally turns to use it for measuring how much, the whole activity is happily rounded off.

Further examples of measuring

In general, measuring out is easier to a beginner than measuring how much. For instance, before measuring a given area using (say) separate inch-squares, a child will naturally build up those inch-squares into areas of various sizes, and can profitably be led to label and examine and compare these shapes.

Again, many older Juniors are interested in the map-measurer, or opisometer; but before using the commercial type, they should be introduced to the rolling wheel as length-measurer through some larger instrument, and should be allowed to make one or two useful wheels of their own. The school may possess a yard-wheel on a trundle, but before using it a child should cut a piece of string to fit its circumference, lay this out along the ground, and roll the wheel along it; he can then check by measurement that the length of the string is one yard. He may wish to mark off a 2-yard or a 5-yard line, and he may make a foot-wheel or ½-yard wheel for himself. Similarly, the opisometer can be used to measure out given lengths; if it is an instrument calibrated with more than one scale, a scale can be selected, a line drawn, and the wheel rolled along a certain stretch that the child can label (say) '5 miles'. This straight stretch can be compared with a 5-mile route on the relevant map.

Important ideas which arise

The work of calibration can give to the child some concept of the linear scale. If the spring is not strained by weights that are too heavy, the graduations of a spring balance for ¼lb., ½lb., ¾lb., 1lb. are evenly spaced; so that he intuitively knows that the 6-oz. mark lies midway between the marks for ¼lb. and ½lb. Here, too, the child is meeting the notion of 'interpolation.' If, besides his measuring work, he is also working on tabulation and graph-making, then these two important concepts—that of the linear scale, and that of inter- and extra-polation—are reinforced and clarified.

A further advantage to children who are used to a variety of graduated instruments is that they become familiar with fractions (and decimals too, if the teacher wishes) in a natural setting. For instance, when a child reads off a measurement as "a half and an eighth", he is giving a worthwhile answer that can be made the basis of a discussion on equivalent fractions.

Altogether, the teacher who encourages the use of a wide range of measuring instruments will find his course enriched and deepened in many ways. Perhaps, among teachers who read this article, there may be some who have experimented in measuring beyond the modest scope I have suggested, or who have taken the work more deeply and obtained findings which would benefit us all. If so, in accordance with the aims of our Association, may we please hear from them?

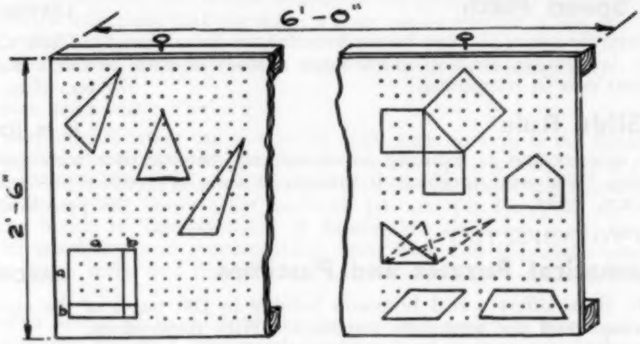
SPLIT PERSONALITY

Headmaster (of a school of 500 pupils) at Morning Assembly: "Now I know that this does not apply to 99·9 per cent of you, but . . . ."

‘GEO’ BOARD EXTENSION

I. B. REYNOLDS AND G. M. DIVALL

A worthwhile acquisition to the wall of any Mathematics room can quickly be constructed, particularly if the school Handicrafts Master’s help can be obtained, by strengthening a 6ft. by 2ft. 6ins. sheet of perforated hardboard with two 2ins. by 1in. softwood battens. The whole apparatus is painted white, providing a background upon which coloured rubber bands, stretched around short lengths of 3/16th″ diameter dowel rod, or special plastic pegs now available for the purpose, are shown in clear relief.



This apparatus approximates a large ‘Geo-Board’—the difference being that, instead of the nails being centralised, the dowels are placed at the ‘corner’ of the squares.

Many visual representations, both algebraic and geometrical, can quickly be constructed; the illustration shows the following e.g.s.

1. $(a + b)^2$
2. Different types of triangles and area of same.
3. Areas of parallelogram, trapezium, triangles on the same base, irregular figures.
4. One visual proof of Pythagoras.

and many further developments can readily be imagined.

In practice, this apparatus has been found to stimulate the pupils, particularly ‘B’ Stream types, because it is ‘big’, ‘robust’, and groups of children can work on it, together, for remedial exercises.

THE FUNCTION CONCEPT

E. T. NORRIS

A report of a talk given to the A.T.A.M. North-western Group at Manchester.

The function concept originated as an idea some fifty years ago, and functional thinking is now regarded as a mode of thought for pupils to acquire. To define it in a sentence it is 'thinking in terms of relationships'.

In the environment everything is changing—e.g. the weather, people, prices—but they do not change in isolation; thus rainfall changes as the pressure of the atmosphere changes, plant growth depends on the amount of fertilizer used, and so on. It is the purpose of science to find how one thing changes with respect to another. Changing factors are called variables by the mathematician; Nunn once wrote that the invention of variables was the most important event in human evolution. Mathematics is needed to describe these relationships, and the emphasis in teaching should be on the essential concepts of variable and dependence. If children were taught to think in terms of relationships, they would not get the idea that mathematics was mainly a matter of techniques.

There is much historical support for the functional approach. The idea was first put forward toward the end of the nineteenth century by the German mathematician Felix Klein: 'The function concept should be the heart and soul of mathematics teaching'. Siddons, some fifty years ago, supported this view by saying that the idea of functionality is the most important that a boy has to acquire in his early mathematics. Again, Nunn's *Teaching of Algebra* may almost be described as a treatise on functionality. But these teachers were ahead of their time (and are still ahead of ours!) in their ideas. The Jeffery Report had the idea of functionality as its basis, while a committee reporting, some ten years ago, on what kind of mathematics should be taught to Sixth Form scientists based their suggested syllabus on the function concept. However, in spite of all this, few teachers and fewer textbook writers have developed the idea.

The function concept finds its place in every type of school, and its use in the classroom should be related to both the symbolic side of mathematics (and this includes number) and the spatial aspect of the subject. Considering the symbolic application first, the idea of a functional relationship may be given and recognized through:

(a) a verbal statement (e.g. when a train goes at a uniform speed the distance travelled depends on the time)

(b) a numerical table (e.g. a list of life assurance premiums for different ages or sums assured)

(c) a graph

(d) a formula

(e) a series

(f) a differential equation.

As the last two belong more to the realms of advanced mathematics they will not be considered here, but some further consideration of graphs and formulae is called for.

On the whole pupils tend to regard a formula solely as a means of computation. Thus they think, for example, of $p_1v_1 = p_2v_2$ as a formula in which to substitute three quantities in order to obtain the fourth, rather than $p = \frac{k}{v}$ as the formula expressing the fact that p varies inversely as v; or of $A = \pi r^2$ solely as a computation formula, rather than the particular way in which one varying quantity depends on another. In other words they do not see the relative importance of the variables in a formula. Thus in the formula stating that the volume of a cylinder depends on height and radius, it is not always recognized that to increase each does not produce the same rate of change of volume. Yet this dynamic use of formulae is the more important as compared with the use of a formula in computation.

A graph is a changing thing, and not, as a rule, merely a number of isolated points joined together; it is necessary to look at the graph as a whole to see the continuously changing aspect. Nunn said that the graph is a mathematical symbol which plays an important part in thinking about the variable. In many schools graphs are taught as an isolated topic, but this is a grave misuse; they should be used throughout the mathematics course to illustrate and develop the basic mathematical ideas. Thus the graph brings out vividly the dependence shown in a formula, e.g. to illustrate how the circumference and area of a circle vary with change of radius, a graphical treatment brings out the essential difference as seen in a linear and a quadratic graph.

Let us consider further the expression of a functional relationship in the form of a verbal statement. Such a statement is relatively imprecise; for example, to say that the area of a circle depends on its radius expresses a relationship, but what sort of dependence is implied? Such a statement is less effective than a formula, which in turn should be studied in conjunction with a graph; formula and graph should be studied together. Thus the formulae

$$\begin{aligned} C &= \pi d \\ s &= 4t \\ A &= 5b \end{aligned}$$

all give straight line graphs through the origin, and all say the same thing: the dependent and independent variables are in the same ratio. This becomes clear from the graph, and the general formula $y = kx$ is obtained. Such an approach to co-ordinate geometry is better than the usual more abstract one of starting with $y = ax + b$ since one is working from the known and concrete.

The next step might be to consider known formulae of the 'square' type, e.g. $A = \pi r^2$; $s = 16t^2$; $H = kI^2$. The graphs reveal the same type of curve, the parabola, while the abstraction of the formula gives $y = kx^2$. The inter-relation of geometrical and algebraic forms (here the parabola and the quadratic function) brings out the co-ordination of the two branches of the subject, and enables one to develop the two aspects side by side.

One of the questions which arises naturally about a graph or formula is how fast or how slow does one quantity change as the other changes, and this leads to a study of rate of change, i.e. an introduction to calculus. Therefore this is not merely a technique, but something needed to enable us to understand more completely the relationship.

The idea of relationship should start in the primary school with number relationships. In the primary school there is too much stress on number facts and not enough on relationships. Thus if a child knows $9 + 9 = 18$, he knows by relationship that $9 + 8 = 17$; since eight is one less than nine, $9 + 8$ is one less than $9 + 9$. Similarly there are many number patterns which the child should learn and recognize e.g. 11, 22, 33, 44, or that nine may be obtained as $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$. If the child be trained in such functional thinking he can then transfer it to other spheres of thought.

To return to the subject of algebra and to summarize what has been said, the emphasis of the work should move away from the technique of solving equations and more significance should be placed on the idea of relationship, including the many different ways in which relationships may be expressed in the world as a whole as well as in the specialized fields such as the biological and physical sciences and in engineering.

The spatial aspect of functionality arises naturally when the continuity of the subject is stressed. The static blackboard drawing is used too much in geometry teaching and simple models made of hinged rods and elastic bring out the essential relationships and properties much more clearly. Thus two rods hinged at one end with their other ends joined by an elastic band may represent a triangle, and as the rods are opened and closed at the hinge the varying aspects of the triangle can be seen as the hinge represents in turn an acute angle, a right angle, and an obtuse angle. It is observed that as the angle changes, the side opposite to it changes in length. Hence one depends on the other. Next, notice the area of the triangle and it will be observed that this depends on the size of the angle also. The statement of the true relationship between these, and—if necessary—a formula, come at a later stage. For the present it is sufficient that the idea of relationship is growing in the child's mind.

The process that has been suggested for a triangle may now be repeated with rods connected to form a parallelogram. For example, as the shape of the figure changes, the relationship of the base angles, and then of the opposite angles may be considered. Statements made at this stage are, of course, general in nature, but such statements as 'equals', 'is greater than', and 'is less then' are all relationships and help the child to develop quantitative ideas. For example, symmetrical patterns show relationships since there is correspondence between one side and another, and between one point and another. Thus, geometrical drawings, shapes, and conditions can be used in ways which bring out *relationships*, thereby making techniques, manipulations and facts intelligible and purposeful to the pupil.

Finally, a glance at the logical basis of the function concept. A. N. Whitehead remarks (or perhaps this is the gist of what he says) that the concept of *relation* is fundamental to human thought. Through the apprehending of relations the mind can go beyond the particular and the immediate, and can learn to appreciate and comprehend the abstract and the remote. (Professor Coulson has a similar point when he says that 'only the pattern gives insight'.).

The late Professor Hamley discusses the basic concepts underlying functionality roughly as follows. Before we can have *relations* there must be elements which can be related. In mathematical logic we start with *classes* or *sets*, and in particular, *variable classes*. When we consider any class or set—for example, a collection of books, or a group of children, or more abstractly a set of numbers—the most important aspect from a mathematical point of view is their *order*. (Not that there is only one order in a set: we can put books in order by authors alphabetically, or by subjects, or by colour, size, etc.). Thus, in our consideration of a set, which involves the idea of a *variable*, we could for instance think of all possible circles, this would be an infinite; set, and one variable involved would be *radius*. Almost inevitably we go on to think of an *ordered* set; and with the set of circles the order would almost certainly be that of increasing radius. But another variable connected with the set of all circles is *area*—again, an ordered set. We can hardly help then but go on to say that as one variable increases, so does the other, i.e. we are thinking of a *correspondence* between one ordered set of variables and another ordered set. Thus we perceive a functional relationship.

So there are four main components in this concept of functionality, all of them fundamental to the logic of mathematics, and therefore the necessary basis of our teaching. They are Set, Order, Variable and Correspondence: and all of these are essential to the idea of Function. We should also note that *continuity* is a special case of the concept of order, and associated with it is the idea of *limit*.

It is essential, therefore, for us to use these components throughout our thinking of, and teaching of number, of symbolic representation, and of spatial relations. Techniques are needed, of course, but they fall into their rightful, secondary place as we put the emphasis on *relationships* and therefore on the ways we explore them and see their applications.

THE USE OF EVERYDAY MATERIALS IN PRIMARY MATHEMATICS

CLAUDE BIRTWISTLE

We tend to accept the everyday commonplace articles of life without giving them any thought or stopping to look at them; the reason we do so is the very fact that they are everyday and commonplace. It is not until someone draws our attention to them that we really see what they are, of what they consist, and what are their dotentialities. Consequently when one looks around for materials for a certain purpose, one often looks too far afield. The mention of mathematical models or mathematical teaching apparatus brings to mind the use of plastics, wood, or perhaps some kind of metal fashioned into a complicated model. This need not be so, and in fact for the average teacher the use of such materials is excluded simply by virtue of the fact that their ability to fashion and work with them is limited or non-existent.

It is then that one should look more closely at materials to hand and seek their mathematical possibilities. In this article I do not propose to write about the mathematical possibilities of measurement in and around the classroom itself, or of the use of classroom shops, household bills and similar day-to-day materials. This has been dealt with fully in an article by Mrs. A. Ogle *Practical Mathematics in the Primary School* which appeared in our issue No. 11 (November 1959), and I would advise all primary school teachers to study carefully the many ideas to be found there. Mrs. Ogle is writing about the mathematics to be obtained by using materials in what one might term their original form. Thus I imagine that a box of processed cheese would appear as something to be sold in her classroom shop. However, I have seen Mrs. Ogle use such a box for a very different purpose: by opening the box and showing the six portions arranged symmetrically and then extracting them one at a time; she had a very good demonstration of fractions (in this case, sixths). Some of the mathematical possibilities of different types of cardboard container were discussed in our last issue by John Lunn in his article *Geometry in the Primary School*.

Containers, in fact, can prove useful in two ways: (a) for measuring capacity or volume; (b) for giving an idea of shape in its various aspects. The use of bottles to give an idea of capacity may be treated in various ways. Medicine bottles are in sizes according to fluid ounces; one can investigate how many fluid ounces in a pint. But better still compare other units: how many pints are there in a cubic foot? or cubic inches in a pint? or pints in a litre? Most children can find the volume of a rectangular box, but which one can suggest a means of finding how many cubic inches of milk he drinks at break? Moreover size is often deceptive when it comes to bottles of different shapes and this could provide material for useful investigation. And what about the 'giant' size of detergent packets? Are they as 'giant' as they are supposed?

The containers themselves present geometrical shapes which can be investigated and described, but it is also useful to discover what plane shapes we get when we cut sections across them or put an elastic band around them to represent a plane cut. To take a simple example, if you have a cardboard box with a square base and you ask what sort of a section you can get by cutting across it, you will probably be told a square, possibly also be told a rectangle, but very rarely will you be told a rhombus or other possibilities such as a pentagon or (if a closed box) a hexagon. Similarly a circular tube cut at an angle will give an elliptical section, but few realize what will

be obtained if a sheet of paper is wrapped around the tube several times before the cut is made, and straightened out afterwards.

Paper and cardboard are, of course, two of the most useful materials in making mathematical models and doing practical work in mathematics. Triangles cut out of paper and folded show quite vividly such things as concurrence of mediums, while repeated folding followed by cutting illustrates axes of symmetry (an article on paper folding is to appear in one of our future issues). Paper cutting can be made to give artistic design, and the delight of children in producing patterns is one which should not be overlooked in the teaching of mathematics.

The production of curves and patterns by drawing envelopes on paper or by stitching curves on thin cardboard is a particular example of the appeal of art through mathematics. Both are being used extensively in primary mathematics teaching nowadays, and probably do not require explanation here. Suffice to say that they are a valuable tool in the gaining of mathematical experience.

Large geometrical shapes (such as triangles, squares, etc.) cut out of coloured cardboard are much more useful and interesting than figures chalked on a board. Once one can move them about, rotate them and turn them over, one is able to compare them and look at them in different ways. They can be attached to the board by drawing pins if desired, or to other flat surfaces using adhesive tape.

Nor need pattern and shape be restricted to plane figures; the construction of solid figures from cardboard is an interesting and useful exercise. The cube and the tetrahedron are easily constructed, and the octahedron is only a little harder. Once these have been produced the urge is to make something more complex, and it is here that one needs to have the nets of the more complex figures in order to be able to cut them and bend them to the required shapes. These are to be found in some geometry books and also in Cundy and Rollett's *Mathematical Models* (O.U.P.). It is possible also to produce some of these solids by using milk straws; such solids have a delightful airy spaciousness, and are delightful to look at quite apart from their mathematical interest.

Milk straws, in fact, are an extremely useful constructional material. Solid models may be constructed by sticking the ends of straws into modelling clay, or simply by twisting the ends of the straws together. But a very effective method of connecting the straws is by threading them together with either string, thread or shirring elastic. Thin string is probably the best where children are making their own models as it can be pushed through the straws, whereas thread and elastic tend to require the use of a needle to 'weight' them through. The simplest straw model to make is, of course, the triangle. Being a rigid structure it does not lend itself to a great deal of mathematical investigation, but if a four-sided figure is constructed (for example, with opposite sides equal) the figure can be moved about on a flat surface and a variety of shapes obtained. Thus, in the example quoted, we obtain a rectangle and a series of parallelograms. And this may be further extended by lifting the straws off the surface and allowing free movement in space. What sort of figures do we obtain now? In the case of the triangle one can start to make solid figures with triangular faces, such as the tetrahedron and then the pyramid. A cube made from straws is particularly interesting as it may be collapsed into a plane figure which seems to bear no relation to the original cube (try it yourself by pushing together two corners which are diagonally opposite to each other.)

Shirring elastic itself (obtainable from drapers and multiple stores) is an extremely useful aid. It can be made into loops and fastened round drawing pins on a blackboard thereby making geometrical figures. By moving the pins one may alter the shapes so that the important 'dynamic' aspect of mathematics is introduced. An article elsewhere in this issue explains the use of peg-board mounted on a wall for a similar purpose. In fact, peg-board and elastic are probably the most effective way of introducing the drawing of graphs. Points are easily 'plotted' by placing pegs in the holes, and curves easily 'drawn' by stretching the elastic round the pegs.

Constructional toys, such as Meccano, have a ready application in mathematical models. Many mechanical models can be made illustrating levers and balances for weighing, but much simpler models can be extremely useful. Gears, for example, are working models of proportion, the rate of proportion being the ratio of the numbers of teeth of the two gears, and if the shafts are turning in the same direction it is direct proportion, whereas when they turn in opposite directions they are in inverse proportion. Another example is provided by fastening two straight Meccano strips together by a bolt acting as a hinge, and joining the other two ends of the strips by elastic, thus forming a triangle. As the strips are moved, the way in which the length of the elastic side changes as the opposite angle changes can be observed, and other similar properties may be demonstrated. If children are encouraged to build mathematical models with such materials, a rich field of discovery of mathematical fact results.

However, it may not always be possible to obtain sufficient constructional material of this type, and a useful substitute for strips such as those mentioned above is provided by lolly sticks. A thousand of these may be bought for a few shillings. They may be hinged by piercing holes near the end, and a useful 'bolt' is provided by gut or nylon bead-thread which can be cut into short lengths, and heated at the ends so that it melts into a globule which acts as a 'head' to the bolt. If one is a little more ambitious in the making of mathematical models, lolly sticks can still be a useful material, while balsa wood (obtainable at most craft stores) is equally useful; its great advantage being that it is easily worked.

It is the trouble of working the various materials which proves a great stumbling block to the ambitions of would be mathematical model makers. Many—especially of the weaker sex!—are put off by the thought of having to use saws, hammers and soldering irons, but if they have a little patience and the will to experiment they will find that plastic provides them with a means of producing some first-class models quite easily. Moreover, the finished article usually has a very attractive appearance. The main point when using plastics is to choose one which can easily be cut by scissors; cellulose acetate sheet is a very useful material in this respect. It is easily cut to any required shape, and to fasten two pieces together it is only necessary to apply acetone with a brush to the edges to be joined, and hold them firmly together for a few seconds until dry. A description of some simple models to make in plastic was given in our issue No. 11.

Modelling clay is a useful material for making small models and has other uses. It has been suggested above as a suitable means of connecting milk straws when making solid models, and a piece of clay spread out on a flat surface is useful when erecting sticks, wires or other articles for use as three-dimensional models. The

clay holds the sticks quite firmly. It may also be used in the teaching of volumes because of its ability to be easily moulded. An inch cube of modelling clay clearly has a volume of one cubic inch, and starting from this one may see the limitless shapes that one cubic inch of clay may take up.

The above are but a few examples of how materials may be adapted to provide mathematical models. There are, of course, many more and most people who are willing to keep their eyes open for suitable materials and to use a little ingenuity should be able to build up a most useful store of home-made models. Moreover, by so doing the teacher will find that some very satisfactory models may be produced at low cost; the most expensive material is that which is made for some specific function, often with a limited market. This is the reason why many commercially produced models appear to be very expensive. An example which illustrates this point occurred recently when the writer had been speaking to some teachers on the use of different types of number apparatus. He was approached by one teacher afterwards who said that he had been using these ideas, but had not been using the commercially produced rods of different lengths. Instead he had visited the local market and bought several lengths of brightly coloured 'poppet' beads. These easily demonstrated the different numbers by lengths, and their addition and subtraction by coupling and uncoupling the beads, "and," he concluded, "you can buy a yard of them for a shilling!"

A visit to Woolworths or a craft shop will give some idea of articles which can be used or easily adapted for the purpose of mathematical materials and constructing mathematical models. Circles may be found as curtain rings or wire circles for the manufacture of lamp shades. Sweet boxes, cans and food cartons provide plenty of cylinders, cubes and prisms. Spheres may be found as marbles, balls, balloons and beads. Nor should shop window displays or other materials be ignored; often such displays contain mathematical shapes and material which the shopkeeper would be happy to pass on when he re-arranges his window. And surely every publican has a stock of cardboard mats from which he could donate a few for use as cardboard discs (with the printing suitably 'blacked out' of course!)

Finally, the teacher should not let his inability to make models deny the class the advantage of learning mathematics through models and similar materials. Firstly, members of a class can make models as a class exercise and in making them learn a great deal of mathematics; also to have his own model enables the pupil to undertake his own mathematical investigation and hence is the ideal. Secondly, if a more complex model is required and the teacher feels incapable of making it, let him (or her) confess it to the class, adding a wistful "If I only had one . . ." It is surprising how often this results in a model (or number of models) being made by the pupils. And even more surprising, it is often the weakest pupil mathematically who makes the best model! There would appear to be a moral somewhere!

May we draw the attention of Primary School teachers to the article appearing in this issue on *Beginnings of Algebra*. Although this appears in our series of articles on Secondary Modern School Mathematics, a great deal of what is covered there applies to the teaching of mathematics at the primary level.

TEACHING AIDS AND LOGIC
III SIMPLE MACHINES WHICH 'THINK'

T. J. FLETCHER

In the last article of this series Servais showed how electrical models can be constructed of the relations which occur in the algebra of sentences—two-valued sentence logic. Here we consider the application of these ideas to the design of electrical circuits which can perform simple logical tasks. We may, for convenience, speak of these machines as 'thinking', but it is obvious that we are only using the word metaphorically. Nevertheless these machines are a challenge to clarify one's ideas on 'thinking', and the phrase 'machines which think' should make one consider exactly how human behaviour differs from that of machines which are designed to simulate it.

The study of these machines is valuable in a mathematical education for several reasons. It provides the pupil with exercises in a new branch of abstract calculation. It gives him an opportunity to see in action the relations between an algebraic theory and a concrete situation. It removes much of the false mystery surrounding 'machines which think', and shows that these machines need be neither elaborate nor expensive. Using the algebra which Boole invented more than one hundred years ago one can design machines to carry our specific tasks, and discuss the behaviour of existing machines under new circumstances. These applications of the algebra were not realised until about 1938. Recently they have become commonplace in American college textbooks, (1,2) but they are taking much longer to catch on in this country. If automation is to become part of our life these ideas should be in every school course.

Before giving examples of circuits which may be constructed cheaply with limited resources we must discuss the notation to employ. When Boole invented his algebra he used a notation as much like that of ordinary algebra as possible. With the passing of the years the fundamental nature of this algebra has been realised and it has become more and more the practice to state it in the independent notation which was used by Servais. There is no doubt that if the time comes when pupils are introduced to these ideas in the Primary school before they have met ordinary algebra, or even any very advanced arithmetic, then one might as well start with what is now the notation used by experts right away. But that is not the situation at the moment, and those who have already been conditioned by traditional algebra might find it more convenient to do Boole algebra as Boole did, and employ his style of notation. This has the advantage that there is the minimum of fresh material to be learnt, and practical calculations can be done with the algebra right away.

Let us recapitulate in this notation. We use a $+$ sign to denote the logical connective '*or* (*or both*)', and a multiplication sign, or simple juxtaposition, to denote the logical '*and*'. A dash is used to denote negation. The following rules then apply:—

$$a + b = b + a \qquad ab = ba,$$

$$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc),$$

$$a(b + c) = ab + ac,$$

$$a + a = a \qquad aa = a,$$

$$(a + b)' = a'b' \qquad (ab)' = a' + b,$$

$$a + ab = a,$$

and there are others also which Servais and the references describe.

Servais showed that in switching circuits '*or*' may be represented by switches in parallel (because a circuit is made if the electricity flows through one *or* the other) and '*and*' by switches in series (because a circuit is made if electricity flows through one *and* the other).

The construction of logical circuits is therefore very simple. One expresses the requirements in *ands* and *ors*, and then codes them by switches in series and in parallel. The algebra provides a convenient short-hand for the whole process, and as one gains experience it can be used to simplify complicated expressions and to carry on logical thinking in the more elaborate situations where common-sense gives up.

The circuits described below can be made using Post Office switches which may be bought for a few shillings. The wiring involved is very simple and well within the resources of any school physics laboratory or workshop. Pupils who like doing things with their hands can build simple chassis and exercise their ingenuity on the many practical problems of layout and design which they will discover for themselves.

SPECIMEN CIRCUITS

A committee consists of four members A, B, C, D who possess three, two, one and one votes respectively. To speed up procedure they wish to instal a system whereby they vote by each one pressing a switch. A light is to light up if the motion is carried. (They vote 'Yes' or 'No', there is no provision for abstention). Design a suitable circuit.

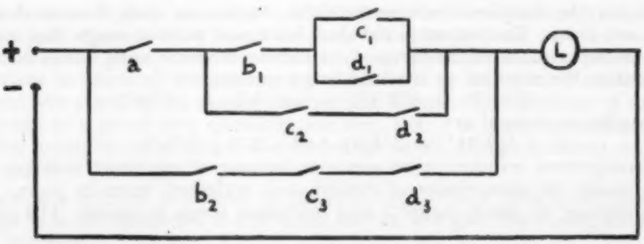
If we denote the proposition 'A votes for the motion' by a, then a little consideration shows that the following combinations of votes in favour will carry the motion:—

$$abcd + abc + abd + acd + ab + bcd.$$

The rules of the algebra, or commonsense, which is only the same thing, reduce this expression to

$$a[b(c + d) + cd] + bcd.$$

We have factorised as much as we can because this saves switches. We now have to wire up a circuit which corresponds.



In this circuit the switches which are all marked with the same letter are ganged together, and for this reason one needs to work with Post Office switches or something similar. These can be obtained with as many as six sets of changeover contacts which work simultaneously. Changeover contacts increase the possibilities in circuit design, and we will deal with these shortly. We adopt the convention of always drawing switches in the off position.

Examples like this are easy to make up, and United Nations sub-committees with their vetoes and blocks who are almost certain to vote together provide many profitable examples.

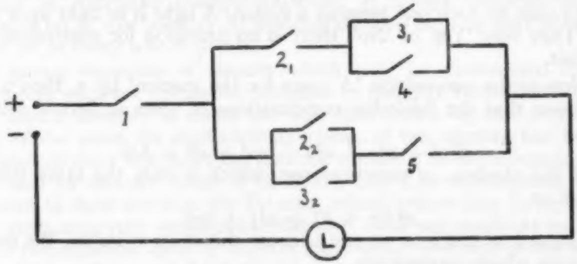
The next example is from the Oxford and Cambridge Schools Examination Regulations. (Quoting from a recent *Mathematical Gazette*) we read:—

Candidates offering History at A level must take Paper I, and two of Papers II, III, IV or V, with the following restrictions:

If IV is taken the third paper must be II;

If V is taken the third paper must be either II or III.

The first point of interest is that these regulations contain a redundancy. This indicates just one of the benefits which a more widespread appreciation of these logical ideas might confer. The interest now is to construct a computor with five switches so arranged that they will light a lamp if a combination which is permitted by the regulations is switched on. Here is one solution:—



The notation should be sufficiently clear.

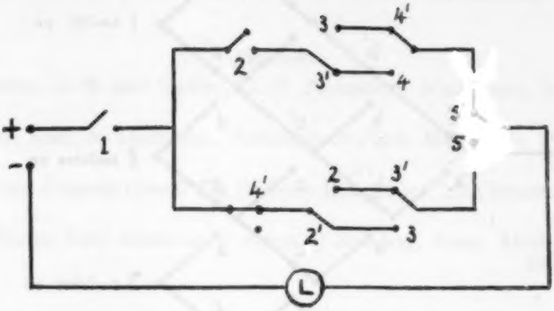
This may be criticised because the light remains on even if more than three switches are down. To overcome the drawback one must arrange that switching on extra switches turns off the lamp. This can be done by using break contacts as well as makes. We must set up the following expression:—

$$1234'5' + 123'45' + 123'4'5 + 12'34'5$$

which may be rearranged as

$$1[2(34' + 3'4)5' + 4'(23' + 2'3)5]$$

This rearrangement economises on contacts because changeoever switches can be used to handle the occurrences of dashed and undashed terms in pairs. Dashed terms correspond to break contacts and undashed terms to makes. The circuit is now

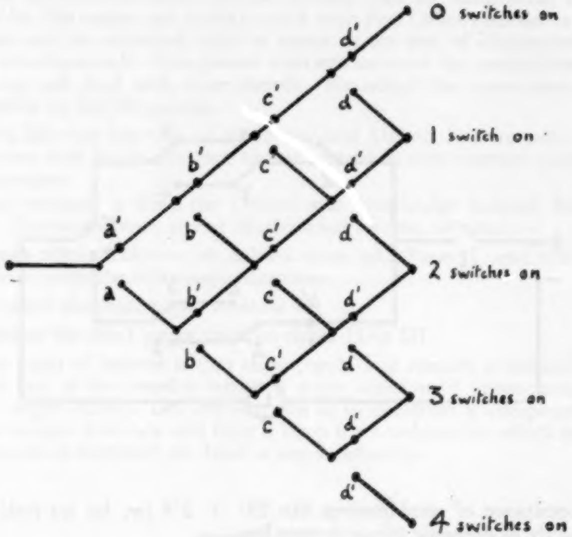


Note the appearance of combinations like $23' + 2'3$ (*or, but not both*) which are wired up exactly as domestic two-way switches.

In this way circuits may be constructed to represent a large number of logical situations. A construction kit with a book containing many such circuits is on sale in America (3). Many of the logical puzzles in the first article of this series may be mimicked by constructing suitable circuits, and games such as Noughts and Crosses present further situations on which to exercise ingenuity. (Incidentally two of my students with no previous experience of electrical circuitry constructed a machine for playing a variant of Noughts and Crosses which was an astonishingly complex piece of wiring. Whilst those who have extensive experience of electrical circuitry may not appreciate the full value of Boole algebra if they come to it late, it undoubtedly helps those with a general mathematical education to attain quite surprising expertise in the field in a very short time).

Our final example is one which should convince even hardened sceptics that practical applications can be found for these ideas, as it deals with an automatic examiner. In some American universities I understand that it is the practice to set examinations in which all the answers are either 'yes' or 'no', and to mark them by machinery. We consider the task of constructing a small scale machine of this kind. The answers to a set of four questions are Yes, Yes, Yes, No. A candidate has to answer the questions by setting four switches on or off. Design a circuit which will light a lamp if he has three or more answers correct.

There is a general procedure for constructing circuits with n switches where the circuit is to be completed if r of them are on. We give the wiring for the case $n = 4$. (overleaf).



Changeover switches are used, and it can be seen that we are merely dealing with Pascal's triangle—but with a Boole algebra interpretation. For particular applications it may only be necessary to use part of this network.

It will be as well to conclude with a brief account of some of the difficulties, as Boole algebra is by no means the complete answer to the switching circuit designer's problems. It will always indicate a method of solving the problem, but it will by no means always indicate the best method; and the reduction of expressions in Boole algebra to their simplest possible form is a problem in which much active research is taking place. Furthermore electrical circuits are not always analysable into components in 'series' and 'parallel.' A Wheatstone Bridge provides a simple example of a circuit whose connections cannot be described merely in these terms. Bridge circuits often provide engineers with short cuts, and Boole algebra, as it stands, is not quite good enough to analyse them as the isomorphism between the algebra and the circuit topology breaks down. We have here yet another case where the typographically simple, linear notations of algebra, powerful as they are, cannot mirror completely all the complexities of a geometrical system. In circuit design, as elsewhere, the need arises to think geometrically as well as algebraically.

Contemporary formalisations of mathematics are nearly all based on Boole algebra and its extensions, and in so far as modern mathematics is a logical structure it rests on these slender foundations. It is relevant to recall that Boole's book was called "The Laws of Thought". But mathematics does not develop by formal

deduction alone, and the boy who wires up a Wheatstone bridge has a wealth beyond the logician's dreams. Electrical circuits, and also mathematical films, suggest that we must seek a new logic based not merely on an algebraic sequence of letters, but on topological interconnections.

The classroom toys of today may show us the new mathematics of tomorrow.

References:

1. Allendoerfer, C. B. and Oakley, C. O. *Principles of Mathematics*, McGraw Hill, 1955.
2. Kemeney, Snell & Thompson, *Introduction to Finite Mathematics*, Prentice Hall, 1957.
3. *The Brainiac Computer Circuits Kit*, Berkeley Enterprises, Inc., Newtonville, Mass., U.S.A.
4. Hohn, Franz, *Some Mathematical Aspects of Switching*, Amer. Math. Month, 62, 1955, pp. 75-90.

CORRECTION

We regret that an error was allowed to appear in the article *Logical Models* appearing in our last issue. The penultimate paragraph on page 16, commencing 'As there are only two positions . . . ' should appear after the diagram at the top of page 17. We apologize to our readers for this mistake.

Also, we have been asked to draw the attention of readers to the fact that the circuit theory appearing in that article is basic to railway signalling systems.

EDITOR.

CORRESPONDENCE

The Editor,
Mathematics Teaching.

Dear Sir,

Regarding Mr. Fletcher's statement (*Mathematics Teaching* No. 12) that the audience of Mathematics Teachers took the film *Rhythmetic* too seriously, I can assure him that the boot is firmly on the other foot. We in the audience were concerned that Mr. Fletcher took it seriously and I am relieved to find that he did not.

Yours, etc.

A. P. HAYNES.

MATHEMATICS IN TRAINING COLLEGES

N. PAYNE

(*We are indebted to the Mathematics Section of the A.T.C.D.E. for permission to publish this summary of plans for the Training College three year course which has been prepared by the Convenor of the Section, Mr. Payne, who is the Senior Mathematics Lecturer of Borough Road College, London*).

Any attempt to give a picture of the impact of the 3-year course of training on mathematics in Training Colleges, immediately brings to light the feature often quoted as one of the many virtues of English education—the freedom of the individual school or college to frame its own curriculum and syllabus of work. There are almost as many different schemes for dealing with the new course as there are Training Colleges. Although each College works within one of eighteen Area Training Organisations (each with its University Department or Institute of Education), the individual college retains considerable freedom to deal with its own particular problems, in its own particular way.

The work in mathematics in any College has two aspects, which may be kept distinct or may be integrated together—the professional work (once called methodology) and the academic work studied by those students offering mathematics as a "Special" subject. The most acute problems in planning the work arise from the diverse mathematical abilities of the students and the wide range of their previous attainment. Many of those following the professional courses will be very weak mathematically and it may be some time since they last studied the subject. Of those studying mathematics as a "Special" subject, previous attainment will vary from a reasonably good pass at G.C.E. A-Level to a bare pass at O-Level, whilst Colleges may also have to cater for students who "dropped" the subject before O-Level. In some Colleges these problems are tackled by providing two or more concurrent courses of different standards. In other Colleges, the problems are tackled by a more extensive use of individual assignments of work, special exercises and tutorials. Some Colleges also make provision for "remedial" classes for those who have previously found difficulty with the subject.

In almost all cases, a student training for Primary School work will follow a course in the teaching of Primary School mathematics, on the grounds that almost all Primary School teachers have to teach the subject. The content of this course will range from the teaching of the traditional Primary School work to a consideration of the place of geometry, algebra, graphical work, statistics, mechanics, astronomy, and other related topics, in the Primary School. Students will usually consider also the impact of recent research on teaching methods, for example the work initiated by Piaget, and the use of apparatus such as the Stern material and the Cuisenaire apparatus.

At some Colleges, courses on the teaching of Secondary School mathematics will be compulsory for all students training for Secondary School work, whilst at others these courses will be optional courses taken mainly by mathematics students. The courses will usually include discussion of the place of practical work, mechanics, statistics, as well as the conventional pure mathematics. At least one College will be providing a special course on the teaching of mathematics in Grammar Schools.

Students choosing mathematics as one of their "special" subjects (they will usually study two or three such subjects), will invariably have a choice of level to which they make take the subject. Terminology for these levels varies, but they are often described as "Subsidiary" Level, "Main Course" Level and "Advanced" Level. The standards reached by students may thus vary from a little below G.C.E. Advanced Level to first and second year university work. Direct comparison is difficult, however, since in addition to more formal work of the traditional type, many courses include work of a more practical nature, for example surveying, astronomy, navigation, much of which will be new to the students.

These different levels will allow colleges to cater for the differing standards of entry of students, and there should be no reason, in the 3-year course, why each student studying mathematics should not carry his study considerably beyond the level he reached at school, both in depth and in breadth. It is essential that students whose previous mathematical education may have been limited to the academic, G.C.E. type mathematics, should be introduced to the more practical type of work which will be of value in the schools in which most of them will teach. Breadth of study is as important as depth, in the training of teachers of mathematics.

Most Colleges will include in their syllabuses some mechanics, some statistics and a little of the history of mathematics. One College is planning a course in the Applications of Mathematics in Science and Industry. Several Colleges will provide courses in Technical Drawing. As examples of the work which "Advanced Level" students will be doing, items listed in the various syllabuses include matrices, groups and sets, projective geometry, differential equations, numerical analysis, vector analysis, the catenary, vibrating strings and so on. Clearly, not all students have the mathematical ability to study the subject to this depth, but the opportunity will be there for those who are capable.

To ensure breadth, many Colleges will provide facility for practical work in such fields as surveying, astronomy, navigation. Students will be able to construct models and apparatus. They will be allowed the opportunity to *discover* their own problems as well as their own solutions.

Methods of assessing the work done by the students, when awarding the final Certificate, also vary from area to area. The two extremes are, on the one hand, final assessment, at the end of the course, by written examination of the traditional type, and on the other, continuous assessment, throughout the course, of written work, practical work, perhaps work in "method" and perhaps a teaching assessment. This continuous assessment method also usually includes the writing of an extended essay on a suitable topic chosen by the student and is often accompanied by an oral examination. Under both methods of assessment, at least one external examiner is appointed to ensure some uniformity of standard within the particular Area Training Organisation.

It will be seen, then, that Training Colleges are taking the opportunity offered by the extension of the course to three years, to deepen and broaden the study of mathematics by the student, at the level which his or her ability warrants. The increasing demands on the teacher by the schools are being kept constantly in mind. Primary School mathematics is no longer limited to arithmetical computation. Increasing numbers of Secondary Modern Schools now provide courses for pupils

up to 16 years of age and over. With the increase in Comprehensive, Bilateral and other multi-range schools, some Training College students find themselves teaching children of above average ability. Finally, an increasing proportion of those Training College students who have specialised in Mathematics or Science are being appointed to the Staffs of Grammar Schools. Complementary to this demand for teachers with a deeper knowledge of mathematics is the increasingly high academic standard of the entrant to the Colleges. An increasing proportion of those specialising in mathematics enter College having pursued a 2 year course in the subject in the 6th. Form. The Colleges are endeavouring to ensure that, whatever stage a student may have reached in his mathematical education, he may progress onwards from that stage, his course satisfying his own need for intellectual stimulus and development, and the schools' need for more qualified teachers.

Those who wish to know more of the thinking underlying the construction of the new syllabuses should consult a report entitled "The Mathematical Ideas and Principles which underlie the Construction of Main Courses in Mathematics", which was prepared recently by the Mathematics Section of the Association of Teachers in Colleges and Departments of Education, and is obtainable (price 1s. 6d.) from A.T.C.D.E. Office, 53A, Brewer Street, London, W.1.

MATHEMATICS IN MARIN

(A Report of the Cuisenaire/Gattegno Approach To Mathematics in Northern California).

ROBIN BRISCOE

All over the United States of America, educators are giving mathematics a close, hard look. What demands come from living in the space age? How can mathematics help create a better life for all human beings as well as meet technical needs? Who needs to learn mathematics? When shall we begin?

These are some of the questions facing all of us concerned with the education, when lasting peace in the world is so closely coupled with survival. We are faced with the importance of educating young children as life-long learners in an ever changing world.

Educators in the three northern districts of Marin County, just north of San Francisco, California, feel that we as teachers must take the first steps by beginning with ourselves. We must again become learners; and we must include work in the important field of mathematics. We must become learners in a new way.

As we ourselves learn, our eyes must be ever more open to seeing what is really involved in learning, so that we may help children learn more effectively. Our work in mathematics must open new doors for us and the children, rather than creating the fear and distrust that many of us experienced in school. A big order? Yes; but we feel we have found the right direction and have taken the first steps.

In February of 1959, a number of us attended a teachers' workshop in Santa Rosa, California, where for several days we worked with Dr. Caleb Gattegno using the Cuisenaire Rods and the Cuisenaire/Gattegno approach to mathematics. It was

exciting! Suddenly, we were not primarily teachers, but learners. We were able to face and work through some of the frustrations of learning where we had so often failed before. We began to sense some ideas about equalities and inequalities. We unlocked a few of the automatically learned and therefore mysterious processes. We began to see that mathematics is not a world of 'Things' but an exciting, fluid world of relationships. And, we watched work with children. We saw how much easier it is for five and six year olds to grasp some of the ideas that have until now evaded us. We saw that the eyes of most young children are free to see what is in front of them, where sometimes adults are not so perceptive.

We became convinced that something important was happening to us and felt that if we could only continue learning ourselves, we could contribute much to helping children learn. Mathematics was one primary concern, but all learning was involved. We felt we were becoming better teachers in the broadest sense.

In order to continue the work, the three northern districts in Marin County joined together: The Dixie School District—Dennie B. Willis, Superintendent; The San Jose School District—Phil Schneider, Superintendent; and The Novato School District—Bill Smith, Superintendent. The local Boards of Education were advised of the project and supported a summer workshop for teachers held in the Dixie District. The doors opened and we began.

Mr. John V. Trivett came to us from Bristol, England, to be our teacher in using the Cuisenaire/Gattegno approach. Originally, he was asked to come to us for one week, with the two following weeks planned for teachers to work on individual interests and class projects. During the first day or two, many of us who had not worked with Dr. Gattegno expressed some of the fears and dislikes frequently attached to mathematics/arithmetic:

"Mathematics! What has that got to do with first grade. We all know what children in the first grade need to learn."

"I never was any good at arithmetic. I can't even balance my own cheque book!"

By the third day, the story had changed to apprehension of a different nature:

"Can't we get John to work with us longer? We're just beginning!"

"We need to see John work with children."

"One week is not enough! Why we'll just get started."

"I wouldn't have believed that I could learn anything about algebra."

"You know, I'm not so dumb after all. I always thought maths was for scientists. I'm beginning to think it is for all of us."

Due to the efforts of the forty teachers in the workshop and the continuing support of the Boards of Education (who had paid not only the original stipend to enable us to work with Mr. Trivett, but the fees for the teachers taking the workshop), we were able to extend our work with John for another week. Visitors began to drop in and stayed to learn.

I must note here, that my report is beginning to sound a bit like much of our American advertising—an easy success story where one merely buys the product and supposedly attains immediate satisfaction. However, nothing could be farther from the truth. Those of you who know the Cuisenaire/Gattegno approach know that for adults, making discoveries and straightening out our tangled web of concepts is no easy job. It is often frustrating and even painful.

But the thrill of real learning, of being able to see for oneself and of being sure that you know, is not trivial. The Cuisenaire/Gattegno approach and Mr. Trivett's help made it possible for us to begin to really learn, and since learning involves an actual change in self, each one of us began to feel the power that comes from added self respect.

At the end of two weeks we were most reluctant to let Mr. Trivett go. We had found a friend as well as a real teacher. People tried to express some of the feelings they were having:

"It is so much easier for the children!"

"When Mr. Trivett's here, I think I can learn, so I do. I wonder if I can teach so that my children will feel this way?"

"I used to think that arithmetic was something in a book. You did the work, closed the book, and that was the end of that. Now I am beginning to think about the ways things and people are related to one another—like families in a way. Mathematics isn't *the people or things*—mathematics expresses the *relationships.*"

"The rods help in a lot of ways. It may sound simple, but they stay there. You can look at them, think awhile and look again. Pretty soon you can tell what you see and if you are right. You don't have to start by keeping ideas in your head. That comes along later."

"I'm sure that I've been telling the children too much. I'm beginning to practice asking questions. If we ask the right questions, the children can make the discoveries for themselves."

"People in Bristol are certainly lucky to have a teacher available. We need to develop ourselves so that we feel more confident about being teachers with the rods."

Here the summer vacation ends early in September. Teachers came back to school eager to work with the Cuisenaire/Gattegno approach and although we did not have any one skilled leader, we found that we had developed enough confidence to try our own wings.

Another workshop was established and coordinated by Miss Robin Briscoe, Curriculum Consultant in the Dixie and San Jose Elementary School Districts, working with Mrs. Dorothy Blackmore from Dominican College, where credit was offered for the work.

Forty-three teachers from the three districts, plus several from other schools in the County planned work sessions and formed sub-groups to facilitate learning. Each Monday afternoon we met from four to six o'clock to exchange ideas, shared the work using the rods with children, and worked out concepts that proved difficult for us. We have felt rather strongly the need for further leadership and are looking forward to the time when Dr. Gattegno will again be in this area, probably in May of 1960.

Plans for this Spring semester include efforts to create teaching-teams using the rods. We feel that we can now make more progress if we can have some common experience with two or more teachers working with a class of children for short periods of time. Work time in the classroom will be followed by time to discuss and examine what we are all learning.

We still feel that we have only scratched the surface of the approach and its values for us. We often wonder what other teachers are doing and feeling. We keep in touch with John Trivett and Dr. Gattegno but we should very much like closer contact with other teachers in England using the Cuisenaire/Gattegno approach and will be glad to answer any letters you might send us.*

Above all, we feel that mathematics is a way of thinking that can be useful and rewarding to human beings of all ages. It can help us perceive, think about and express relationships which are crucial to living. We believe that this broad basic approach to mathematics must be begun with young children, as well as with those of us who are older, so that children can be helped to maintain the spontaneity and develop the self respect they need to become fully contributing and receptive members of the human race.

* If you would like to correspond with a teacher here, please write. If you send the letter to me with an indication of the person you might like to hear from (i.e.—a fifth grade teacher, an administrator of an elementary school, etc.) I'll see that your letter gets to an appropriate person.
Address: Miss Robin Briscoe, Curriculum Consultant,
Dixie School District, 4775 Redwood Highway,
Marin County, California, U.S.A.

SEVENTH ANNUAL GENERAL MEETING

At the meeting on March 19th, 1960, at Redman's Park Hotel, Blackpool, the Secretary reported the formation of four new Branches of the Association in the Cambridge area, East Suffolk, Bristol and the North-East (Newcastle).

The Treasurer, in presenting the accounts for the past year, was able to show a smaller deficit on the year's working than in the previous year. She hoped that this would be cleared completely by the end of 1960. (A summary of the audited accounts is printed after this report).

The Conference Secretary and the Editor drew attention to the need for more lecturers, demonstrators and writers to help meet the increasing demand for advice about mathematics teaching in primary schools.

The completion by the Film Unit of 'Tangency' and 'Four Point Conics' was announced.

A list of officers and committee members appears on the back cover of this issue.

D.H.W.

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

INCOME and EXPENDITURE ACCOUNT FOR YEAR ENDING DEC. 31st, 1959

EXPENDITURE	£ s. d.	£ s. d.
To: Reprints from Bulletin No. 8		3 12 0
Printing and Distribution Bulletin No. 9	141 15 6	
Printing and Distribution Bulletin No. 10	109 8 7	
Printing and Distribution Bulletin No. 11	125 16 8	
Amt. due and unpaid Dec. 31st. Bulletin No. 11	2 14 0	
		379 14 9
Printing of Pamphlets	3 14 11	
Amt. due and unpaid Dec. 31st. Pamphlets	16 5 0	
		19 19 11
Hon. Secretary	42 15 8	
Hon. Treasurer	10 0 0	
Editor	23 0 0	
		75 15 8
Committee Travelling		49 6 7
Printing, Stationery		18 15 1
Bank Charges	5 11 9	
Cheque Book	4 2	
		5 15 11
BALANCE, being Excess of Income over Expenditure		69 12 9
		622 12 8

INCOME	£ s. d.	£ s. d.
By: Subs.		
1959 paid in 1958, 92 @ 10/-	46 0 0	
1958 .. .. .. 15 @ 5/-	3 15 0	
1959 .. .. a) 717 @ 10/-	358 10 0	
b) Bankers Order 101½	50 15 0	
FOREIGN — 1959	9 18 8	
		468 18 8
Sale of Bulletins		24 9 6
Sale of Pamphlets		13 2
Adverts in Bulletins	33 7 0	
Adverts in Bulletin 11, due Dec. 31st and unpaid	19 10 0	
		52 17 0
Meeting–Birmingham—February Gross Receipts £96 7 0 Payments £76 10 0		19 17 0
Donations:		
a) Mrs. R. M. Fyfe	25 0 0	
b) N. W. Branch A.T.A.M.	15 0 0	
c) Cambridge Inst. of Educ.	10 10 0	
d) Other Donations	5 7 4	
		55 17 4
		622 12 8

BALANCE SHEET AS AT DEC. 31st, 1959

LIABILITIES	£ s. d.	£ s. d.
To: Subs. prepaid 1960 paid in 1958	5 15 0	
1960 paid in 1959	81 10 0	
1961 paid in 1959	10 0	
1961 paid in 1959	2 15 0	
1962 paid in 1958	10 0	
		91 0 0
Sundry Creditors Bulletin No. 11	2 14 0	
Pamphlets 3 and 4	16 5 0	
		18 19 0
		109 19 0

ASSETS	£ s. d.	£ s. d.
By: Cash at Bank		19 1 3
Sundry Debtors—Adverts		19 10 0
Deficit 31.12.58	141 0 6	
Less Surplus 1959	69 12 9	
		71 7 9
		109 19 0

PRESIDENTIAL MESSAGE

I would first say how greatly I appreciate the honour which the Association has done me in inviting me to be its president. This is a critical time in our affairs as the original founders of the Association no longer remain in executive office and we are now led by a committee which is largely of the "second generation." The Association is now very large and the time when one could hope to know the majority of members is long past. I hope that the temptations to relax into a new orthodoxy will be strongly resisted, and that we will always be working on ideas which are new and challenging.

For the first few years of our growth we had sufficient unfamiliar ideas to provide momentum, but the responsibility of maintaining a large association means that many of our most energetic members now have less time for original work, and unless much more hard research on the comparatively untouched problems of mathematics teaching takes place we will grind to a standstill. The new committee arrangements should release some of our more creative minds from administration, and it is to be hoped that we can develop a system whereby a steady stream of new recruits from local branches can graduate to a spell of duty on the main committee; and then move on, while still young, to the rank of "elder statesmen" where, knowing the aims of the Association and also the practical details of its running, they can produce original research in the teaching of our subject which is passed on to our members.

Our great need is fresh ideas, and deeper insight. Many teachers now join us not because of what they are prepared to give, but for what they think we can tell them; but it should not be our task to codify what is best in current practice or to produce agreed reports which are acceptable to as many people as possible, we should seek the orthodoxy of tomorrow which is heresy today. We need an inner cell within the Association which is seeking new discoveries all the time. Every year sees amazing advances in mathematics itself, but few, if any, advances in communicating it to the population at large. The communication of mathematics to other people is as big a research problem as any which the subject has to offer.

In my presidential year I hope to see as much of the Association's activities as I can, but above all I would wish to increase our reaserch effort. I will be happy to discuss with anyone ways and means of finding out more about teaching our subject. In the International Commission for the Study and Improvement of the Teaching of Mathematics I find a happy few who seem to me to approach these problems on sound lines—not studying teaching at the "I always do it like this" level, but by searching more deeply in the borderlands where mathematics meets psychology and philosophy. The difficulties which many pupils have with our subject are emotional, and they cannot be overcome by changing syllabuses or writing textbooks; they may be overcome by understanding the mainsprings of human action and mobilising the pupil's entire energies for the task.

The role of teaching aids in this programme is not merely that of providing supplementary illustrations—a voluntary extra; it is to show the origin of mathematics in experience, the origin of theory in practice. I find these ideas understood and discussed among the International Commission when I seek almost in vain for similar understanding and discussion in Britain. But then many members of the International Commission come from countries whose philosophical traditions differ from our own. In a highly critical, experimental approach to our classroom work, with a determined effort to base it on more philosophical foundations and in consultation with the best opinion from abroad we may find a way to salvation; but the way cannot be organised by committees, it can only be sought by individuals.

In my year of office and in the years to follow I invite you, personally, to join me in this quest.

TREVOR FLETCHER.

BOOK REVIEWS

Primary Mathematics

An introduction to the Language of Number. Flavell & Wakelam. Methuen.
Pupil's Book I pp. 128 5s. 6d. Teacher's Book I pp. 44 6s.

These books are the first of a series of three which are written from the 'new point of view' in primary mathematics teaching. An experienced teacher will find them very unfamiliar. Length, weight and volume and their estimation, right angles, rectangles, halves and quarters (geometrically as well as using all the conventional applications), number stories, square numbers, filling in gaps in sequences, use of number rods (Cuisenaire material ?), nail (Geo- ?) boards, the compass points and the four rules applied to two figure numbers: all these make up a bill of fare which the first classes ought to find stimulating and which should lead on to a well developed *mathematical* treatment of primary school mathematics.

Many teachers have been looking for a textbook which goes a long way toward meeting the needs of the principles illustrated by A.T.A.M. lectures and demonstrations. They will find in this book a sound foundation on which further developments may be based. The Teacher's book backs up the children's with some very sound common sense and hints on teaching.

The books might have used colour to make their format more attractive. The printing is bold and clear and the work is simple if only the teacher will provide the necessary materials. C.H.

Arithmetic with Numbers in Colour

C. Gattegno Heinemann Book 1. 44 p. Book 2. 80 pp. 3s. each.

It comes as a shock to many teachers when they are shown the examples given in these books which are intended for beginners at arithmetic. $\frac{1}{2} \times (\frac{1}{4} \times 24)$ seems beyond the imagined capabilities of most children below the age of 8 and that they should comprehend the meaning of this, seems incredible.

Our association has done much to publicise the use of the Cuisenaire material and we have shown with other teachers' children that the claims are not unjustified. These books cover the four rules of number using this material and in addition, because it is natural to say that if two rods are equivalent to a third then each of the smaller rods is a half of the larger, simple multiplication of a whole number by a fraction is learnt alongside the other processes as a different way of saying something about the rods.

Many teachers will prefer to experiment with children and to proceed on the basis of what their children discover. There are, however, a large number of teachers who feel the need for guidance. Book One outlines a sequence of activities which a teacher might suggest to children and the book might be given to them as their reading ability develops and as a way of helping reading skills alongside an interesting mathematical activity. The book is intended to ensure that a full basis of experience is provided before a more systematic attack is launched. The material makes it possible to restrict the number sizes considered so that Book One covers the numbers from one to ten and includes a treatment of the meaning of fraction e.g. 7/9ths.

Book Two covers in two sections, numbers from 1 to 20 and numbers greater than 20 as far as short division.

There is no need to follow the sequence in the course but a small group of able children will be enabled to progress rapidly by the use of these books. Many teachers will bless Dr. Gattegno for providing these courses. Many people thinking about the rearrangement of topics in the primary school may be encouraged by this revolutionary treatment. It is an example of what may be achieved when symbols are used not as objects for manipulation but as a means of describing the physical activities with concrete apparatus.

If you are, or are thinking of, using Cuisenaire material, you should get these two books.

A Simple Approach to Electronic Computers

E. H. W. Hersee. Blackie and Son. pp. 104 12s. 6d.

This is a very good little book for the uninitiated. Its scope is amazing, covering the simple mathematical principles of the method of operation and of programming both digital and analogue computers without referring to any particular make of machine.

After a brief but lucid treatment of number representation in digital computers, follows a very simple description of memory, control, input and output which includes a very simple non-physical description of the principles of the basic circuits. The section continues with an account of how programming is carried out, simplified by the omission of coding. This section closes with a very brief account of special purpose digital computers. Perhaps the interest of the reader would have been even more stimulated, had this section been expanded.

The last section deals with analogue computers, again trying to deal with principles rather than with particular machines. A very simple problem involving a second order differential equation is programmed by way of illustration of the use of these machines.

The book is a clearly written introduction to the more learned treatises on the subject and would give a student who had ideas of taking up mathematics as a career, information about a very important branch which has come to the fore in recent times. The book is strongly recommended for school libraries in all types of secondary school. C.H.

An Introductory Course in Pure Mathematics

K. B. Swaine Harrap pp. 288 15s.

This book provides a sixth form course in pure mathematics to advanced level and is intended for use in class rather than for private study. It is refreshing in its approach since the work covered appears as a unified course springing from a core of analytical geometry. The book-work has been reduced to a minimum and the general pattern appears as a series of exercises, but through these the student has the

opportunity of discovering for himself many of the milestones in pure mathematics. Not only is the way laid open for the usual lines of development, but there is sufficient stimulus to make the student with an enquiring mind want to investigate further the topic in hand.

The section on the theory of quadratic functions leads quite naturally into the theory of rational functions and the resulting curve sketching gives some interesting opportunities for investigating the properties which create the sometimes unusual appearance of the graph of a function. Here, as at every stage, the subtle use of leading questions helps the student to make the maximum use of the tools of his background and also signposts the way to further development.

This is a book which gives plenty of scope to the abler student as well as catering for the needs of the less able and it could be used to good advantage with any sixth form class. I. L. Campbell

Pure Mathematics—A First Course

J. K. Backhouse and S. P. T. Houldsworth Longmans pp. 472

This book is intended to give a one-year introduction to pure mathematics and covers all the necessary groundwork in the normal sixth form course. It is well-written and the clear book-work should prove helpful to the individual student, particularly as each section is followed by a more than adequate number of examples.

The section dealing with trigonometric ratios is particularly good but the treatment of logarithms is disappointingly thin. On the whole one has the impression that, although the development of each topic is logical and not lacking in rigour, they are only slightly related to each other. It is probable that the division between the chapters is rather too rigid to give the student the over-all picture that would be desirable in laying the foundations of pure mathematics.

This would be a useful book to have in a school library and would be a good textbook for any student who is working individually. I. L. Campbell

Exposé moderne des mathématiques élémentaires.

Lucienne Felix. Dunod, Paris pp. 421

The reviewer is not often faced with a task as difficult as this; and all those who take the view that "we know what mathematics is, and we just have to go in and tell them" should read this book. Miss Felix, a teacher of great experience who is held by her French colleagues in high esteem. has written a book completely without compromise, presenting school mathematics in a fashion so modern that most graduates would stagger under its impact. The book is intended for use in the upper forms of grammar schools; it reviews school work from an advanced standpoint and equips the student for the University career which is to follow. A school text can never have been written which was less of "the mixture as before," as it is excellent

where other books make no attempts to go at all, and it is weak in places where we take a good performance for granted. It will not prepare a candidate for any examination I know—and this I regard as an outstanding merit. It could be described as an up to date version of Klein's "Elementary Mathematics from an advanced Standpoint," but it is written for a rather different public and has a different aim, being a school text written in the tide of enthusiasm for the new outlook which the Bourbaki movement has produced in France.

The central aim is the unification of school mathematics, and this is done with a Gallic love of system, rigour and intellectual discipline much at variance with English expediency and pragmatism. As pupils read this book they must develop the pure mathematician's desire to prove the obvious, to *prove* what they think they *know* already.

The approach is axiomatic. The first section deals with fundamental structures, symbols, vector spaces and sets. Arithmetic, algebra and analysis follow. Trigonometry is based on a foundation of vector spaces; and then comes a section on geometries, with the word in the plural because non-Euclidean geometry is discussed on equal terms with Euclidean. The geometry we knew at school has almost disappeared as there is no interest in a collection of anecdotal results, but the emphasis is all the time on the big theme of transformations with a group structure. Affine geometry and projective geometry precede metrical geometry, and the introduction of 'angle' is a matter of some complexity. In addition the attitude to geometry is the hard one—synthetic rather than analytic.

It can be seen that the book demands so much from the reader that many teachers would abandon it entirely; but it cannot be ignored as it is the vanguard of an invading army whose advance not even the entrenched forces of English habits will be strong enough to stop.

In a text of such wide scope it is only natural that there are many details which one would wish to alter, but these minor matters are not our present concern. However, as a school text the book has serious drawbacks, a major one being that there are very few examples and no problems at all for the pupil to work. The young mathematician must cultivate manipulative skill as well as the broad philosophical understanding which is the aim here, but without an accompanying set of exercises one could make no progress at all, and what is one to use? Furthermore, Miss Felix is well aware of the creative role of informal heuristic reasoning, and the need to develop the pupil's imagination by building on the foundation of his experience, but her book is not concerned with these aspects of teaching; but unless a teacher using it is so concerned the results could easily be a disastrous servitude to a new formal dogmatism, worse than that associated with some teaching of the past, precisely because these new abstract structures are more fundamental and far-reaching in their power.

I cannot feel that a book like this could be of much direct use in our schools for many decades, but I hope that it will be studied by Training College teachers with the scrupulous care and attention which it deserves; for there is nothing comparable in our own language and it deals with changes which we have to meet, which we could ignore but which we cannot reverse. T. J. FLETCHER

APPARATUS AND MATERIALS REVIEW

Tangency

16 mm. silent film (black and white) produced by I. Harris for A.T.A.M. Film Unit. 8 min. Price £8 8s. per copy, or on hire 5s. per day (plus postage) from 122, North Road, Dartford, Kent.

The film is a teaching aid of limited flexibility; whatever one does, the director of the film must impose himself between the teacher and his pupils. One has to see things in the order, and very largely in the manner, which he decrees. In this film Mr. Harris has tried to make his presence felt as little as possible; there are no titles or sound track, and the pupil is left to observe and draw his conclusions. Consequently this film may be used at many different levels of ability and in many different ways.

The circle appears first as a locus, and a straight line then moves across the circle, remaining parallel to itself. The line cuts the circle in some positions, not in others, and in one position gives the tangent. Next the film considers the points of intersection of the line with the circle, and the locus of the mid-point of the intercept of the circle on the line. We next discover the tangent from a straight line rotating about a point on the circumference of the circle, and follow this with successive magnifications of a part of the line and circle to give the idea of the tangent as a limit. Finally we see the circle introduced as an envelope and the film ends with the circle rolling away on a tangent.

The teachers who use films in their teaching will quickly incorporate this one into their lessons; those who have never used this medium will find this film one of the best to experiment with.

Elastic Band Geometry

Nailboards; plastic pegs. Mathematical Pie Ltd., 97, Chequer Road, Doncaster.

These nailboards are made from 9 mm. birch plywood with brass round-headed nails. The boards are lacquered and in use were found to be ideal as far as visibility and not collecting dirt were concerned. The price, including a supply of elastic bands and a leaflet on the use of the boards, varies with the size (e.g. 9 in. square with 36 nails, 4s. 6d.), but the price is reasonable, although the teacher-handyman could produce his own much more cheaply, of course.

If price is a major consideration, the same firm sell plastic pegs in various colours for use with ordinary peg-board. The latter may be bought at handyman's stores quite cheaply and cut into small pieces. The pegs fit firmly into the board and allow elastic bands to be fastened around them. They also have a hole in the top which allows a screw hook to be inserted if desired. The pegs are excellent and also cheap at 9s. 6d. per gross or 11d. per dozen (postage extra).

C.B.

Since our last issue, in which we reviewed the *Structural Arithmetic Apparatus*, a small booklet has been published by E.S.A. which might be of use to those interested in this material. It is called *Experiments with Structural Arithmetic in an English School* and costs 2s. 6d.

A.T.A.M. DAIRY

July 2nd. Cambridge Area Group Meeting at County Primary School, Waterbeach, Cambs., commencing 9-45 a.m. "Division of Number" by formal methods and by Cuisenaire. Details: N. Reed.

October 7th/8th At Woodlands Secondary School, Basildon, Essex. "Secondary School Mathematics". Details: A. Ivell.

October 8th. North-East Group Meeting in Newcastle. Details: B. O'Byrne.

October 15th. North-West Area Meeting in School of Education, Dover Street, Manchester. "From Primary, through Secondary to Training College." Details: Miss D. Clutten.

November 19th. Middlesex Group. Lectures and Exhibition of Aids. Details in our next issue.

Autumn meeting of East Suffolk Group to be devoted to films and filmstrips. Details: A. S. Hampton-Wright.

For addresses of Local Secretaries see Back Cover.

A.G.M. 1961.

Our next A.G.M. will take the form of a four day Conference in London in April. A programme of lectures, visits and discussions is being arranged. This is an opportunity for members to meet each other, to keep in touch with the work of the Association, and to help to shape future policy. Residential accommodation is being booked and the final dates decided as we go to press; it will be during the week of April 9th to 15th. Full details will be given in our next issue.

BACK NUMBERS OF MATHEMATICS TEACHING

Copies of back numbers of this magazine may be obtained from the Secretary. At present there are still copies available of issue No. 5 (November 1957) and all subsequent issues. The price of No. 5 is three shillings, numbers 6 to 8 cost 2s. 6d. each, and numbers 9 to 12 cost 3s. 6d. each.

CHANGE OF ADDRESS

Members are asked to notify the Hon. Treasurer immediately should they change their address. The Association regrets it cannot be responsible for non-delivery of 'Mathematics Teaching' where this is due to failure to notify such change.

STOP PRESS—A.G.M., April 11th to 14th, 1961
Reserve these dates!

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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